

Modeling Farm Households' Price Responses in the Presence of Transaction Costs and Heterogeneity in Labor Markets

Christian H.C.A. Henning and Arne Henningsen

Department of Agricultural Economics, University of Kiel
Olshausenstr. 40, 24098 Kiel, Germany, [http://www.uni-kiel.de/agrarpol/
chenning@agric-econ.uni-kiel.de](http://www.uni-kiel.de/agrarpol/chenning@agric-econ.uni-kiel.de), ahenningsen@agric-econ.uni-kiel.de

This is an electronic version of an Article published in
American Journal of Agricultural Economics, 89(3), p. 665-681.

©2007 American Agricultural Economics Association.

The final version is available on <http://www.blackwell-synergy.com/loi/AJAE>

Abstract

We develop a farm household model to analyze price responses of farm households. This model incorporates various types of transaction costs as well as labor heterogeneity. Non-proportional variable transaction costs or labor heterogeneity imply that production and consumption decisions become non-separable, even when the household buys or sells labor. An empirical model is estimated using data from Mid-West Poland. The results show that non-proportional variable transaction costs and labor heterogeneity significantly influence household behavior. Not all price elasticities, however, change significantly if these are neglected.

Key words: farm household model, market imperfection, rural labor markets, selectivity, transaction costs

JEL classification: Q12, J22, D43, C51

Christian H.C.A. Henning is professor and Arne Henningsen is assistant professor, Department of Agricultural Economics, University of Kiel, Germany. Senior authorship is shared. The authors thank Wade Brorsen and three anonymous reviewers for their very helpful comments and suggestions. We also thank Awudu Abdulai, Uwe Latacz-Lohmann, and Jerzy Michalek for very helpful comments. Any remaining errors are the authors'.

The agricultural development literature has long recognized that rural markets are often underdeveloped or absent. These market imperfections create transaction costs and, if transaction costs are sufficiently high, households find it unprofitable to either buy or sell a good in the market, i.e. remain autarkic (de Janvry, Fafchamps, and Sadoulet 1991). In this case, production and consumption decisions are no longer separable and conventional microeconomic theory is no longer suitable to model farm household behavior. As a result, farm household models (FHM) have been developed that explicitly incorporate the interdependency of production and consumption decisions.

Early FHM studies use non-separable FHMs to explain sometimes paradoxical — and even perverse — microeconomic responses of peasants to changes in relative prices (Strauss 1986; Lopez 1984; de Janvry, Fafchamps, and Sadoulet 1991; de Janvry et al. 1992). Several theoretical and empirical studies have used the FHM approach to analyze farm household responses under imperfect labor (Lopez 1986; Thijssen 1988; Benjamin 1992; Jacoby 1993; Sadoulet, de Janvry, and Benjamin 1998), capital (de Janvry et al. 1992), or food markets (de Janvry, Fafchamps, and Sadoulet 1991; Goetz 1992; Omamo 1998; Skoufias 1994; Abdulai and Delgado 1999). However, non-separability makes theoretical and, in particular, empirical analyses more difficult. Therefore, most empirical analyses assume separable FHMs or use reduced forms of a non-separable FHM.

In contrast to early FHM work, recent studies emphasize transaction costs and institutions in determining households' decisions on market participation (Goetz 1992; Key, Sadoulet, and de Janvry 2000; Vakis, Sadoulet, and de Janvry 2003; Vance and Geoghegan 2004; Carter and Yao 2002; Carter and Olinto 2003). For instance, Key, Sadoulet, and de Janvry (2000) develop a model of supply response when transaction costs cause some producers to buy, others to sell, and others not to participate in markets (Key, Sadoulet, and de Janvry 2000, p. 245). They consider fixed transaction costs (FTC) and proportional transaction costs (PTC) only. Fixed transaction costs are invariant to the quantity of the good traded, whereas proportional transaction costs increase proportionally in quantity. Thus, PTC correspond to constant marginal transaction costs.

An aspect that is conceivable, but has not yet received attention in the FHM literature is the role of non-proportional variable transaction costs (NTC) on production and consumption decisions or market participation. We fill this gap by examining how NTC affect farm household decisions.

We also show that not only transaction costs, which are partly implied by unobserved heterogeneity, but also observed heterogeneity of labor can result in a non-separable FHM. To this end, we construct an FHM, taking into account labor market imperfections via FTC, PTC, NTC, and observed labor heterogeneity. Based on this generalized FHM approach, we derive the following theoretical results: (i) non-separability of production and consumption decisions

can occur even if households participate in markets, (ii) imperfect labor markets take a middle ground, with respect to price responses, between standard non-separable FHMs assuming absent labor markets and standard separable FHMs assuming perfect labor markets, and (iii) a test of the joint significance of NTC and heterogeneity for farm household's behavior is possible.

We estimate our generalized FHM approach econometrically using farm household data from Poland. The estimation procedure utilized allows us to consider both potential selectivity and endogeneity problems.

Furthermore, we explicitly test for the significance of NTC and heterogeneity in rural labor markets as well as for the differences between price elasticities calculated for different degrees of labor market imperfection.

Theoretical Model

In this section we construct a static model of the price responses of farm households in imperfect and perfect labor markets (see also [Glauben, Henning, and Henningsen 2003](#)). To concentrate on the role of labor market constraints, our model ignores some aspects of farmers' decisions, notably (price) risk ([Finkelshtain and Chalfant 1991](#); [Fafchamps 1992](#)) and credit constraints ([Chambers and Lopez 1987](#)). The farm household is assumed to maximize utility subject to a technology, time, and budget constraint. Therefore, farm households solve the following maximization problem:

$$(1) \quad \max_{\mathbf{x}, \mathbf{c}} U(\mathbf{c})$$

subject to

$$(2) \quad G(\mathbf{x}, \mathbf{r}) = 0 \quad (\text{production function})$$

$$(3) \quad T_L - |X_L| + X_L^h - X_L^s - C_L \geq 0 \quad (\text{time constraint})$$

$$(4) \quad P_m C_m \leq P_c X_c + P_a (X_a - C_a) - P_v |X_v| - g(X_L^h) + f(X_L^s) + E \quad (\text{budget constraint})$$

where $U(\mathbf{c})$ is the farm household's utility function, which is monotonically increasing and strictly quasi-concave, and \mathbf{c} is a vector of consumption goods consisting of market commodities (C_m), self-produced agricultural goods (C_a), and leisure (C_L).

Production technology is represented by a well-behaved multi-input multi-output production function (2) ([Lau 1978a](#)), where \mathbf{x} is a vector of production goods, expressed as netputs, and \mathbf{r} is a vector of quasi-fixed factors. The farm household produces pure market goods ($X_c > 0$) and goods that are partly consumed by the household ($X_a > 0$). It uses variable intermediate inputs ($X_v < 0$), labor ($X_L < 0$), and the quasi-fixed factors land (R_g) and capital (R_k).

The farm household faces a time constraint (3), where T_L denotes total time available. $|X_L| = X_L^f + X_L^h$ is the total of on-farm labor time subdivided into family labor (X_L^f) and hired labor (X_L^h), and X_L^s denotes off-farm family labor. There are four possible regimes of labor

market participation. First, the household simultaneously sells family labor and hires labor. Second, farmers neither sell nor hire labor (autarky). Third, households only sell off-farm labor and fourth, they only hire on-farm labor. Earlier studies have neglected the regime in which households simultaneously hire and supply labor. For instance, [Sadoulet, de Janvry, and Benjamin \(1998\)](#) argue that this labor market regime is rarely observed and that their theoretical model cannot explain this specific labor strategy. However, in our data set this regime is rather frequent, with 29% of households falling into that category (table 1).¹

The budget constraint (4) states that a household’s consumption expenditures (left-hand side) must not exceed its monetary income (right-hand side). The household may receive income from farming and off-farm employment. In addition, it receives ($E > 0$) or pays ($E < 0$) transfers, which are determined exogenously. Here, P_i , $i \in \{m, a, c, v\}$, denote the exogenous consumer and producer prices.

Rural labor markets are often plagued by incomplete formal institutions, which implies transaction costs ([Benjamin 1992](#); [Sadoulet, de Janvry, and Benjamin 1998](#); [Key, Sadoulet, and de Janvry 2000](#)). Transaction costs are normally considered as FTC and PTC in existing studies ([Key, Sadoulet, and de Janvry 2000](#); [Vakis, Sadoulet, and de Janvry 2003](#)). In particular, PTC correspond to transportation and marketing costs, while search, information, negotiation, and bargaining costs as well as screening, enforcement, and supervision costs are generally considered as FTC ([Key, Sadoulet, and de Janvry 2000](#)). Although the concept of FTC and PTC appears intuitive, there is apparently no theoretical justification for excluding NTC ex ante. Empirically there might be some transaction costs that vary non-proportionally with the quantity traded, implying NTC for both on-farm labor demand and off-farm labor supply. Theoretically, it is unclear how the marginal costs vary, i.e. if they are increasing, decreasing, or constant.² In this article we present a theoretical framework that considers the impact of NTC on both on-farm labor demand and off-farm labor supply, and also provide an empirical test of their significance.

To formally include NTC as well as FTC and PTC in our model, we denote total variable transaction costs (PTC + NTC) of off-farm employment by $TC_v^s(X_L^s, \mathbf{z}_v^s)$ and total variable transaction costs of on-farm labor demand by $TC_v^h(X_L^h, \mathbf{z}_v^h)$, where \mathbf{z}_v^s and \mathbf{z}_v^h denote factors explaining variable transaction costs of the farm household for selling and buying labor, respectively (see

¹ Simultaneously hiring on-farm labor and supplying off-farm labor can be rational with a strictly convex labor cost and a strictly concave labor income function. For instance, if the skills of the household members to work off-farm are very heterogeneous, it is rational to simultaneously supply high-priced labor of well-educated household members and hire cheap agricultural labor (see also [Sadoulet, de Janvry, and Benjamin 1996](#)). A more detailed explanation is provided in [Henning and Henningsen \(2007\)](#).

² We do not intend to provide a comprehensive theory of rural labor market organization and its impact on transaction costs, but rather leave it to future research. Some intuitive examples of NTC are however provided in [Henning and Henningsen \(2007\)](#).

Key, Sadoulet, and de Janvry 2000). For the special case of only PTC these functions are linear in X_L^s and X_L^h , respectively.

Transaction costs are partly implied by unobserved heterogeneity of labor (Spence 1976; Eswaran and Kotwal 1986; Frisvold 1994; Sadoulet, de Janvry, and Benjamin 1998). However, heterogeneity of labor quality might also have an impact, although it can be observed by employers. For example, family members might have heterogeneous skills to work off-farm, which are generally observable by firms. In such cases, family members would receive different off-farm wage rates corresponding to their observable skills.

If we further assume that family labor is homogeneous regarding farm work, profit maximization implies that the order in which family members work off-farm corresponds to their skill levels, further implying that marginal off-farm wage is a step-wise decreasing function of off-farm labor supply. We approximate the step-wise labor wage function by a continuous function. Subtracting marginal transaction costs, we obtain the following effective marginal labor wage function:

$$(5) \quad P_L^s = \bar{P}_L + b^s(X_L^s, \mathbf{z}_L^s) - \frac{\partial TC_v^s(X_L^s, \mathbf{z}_v^s)}{\partial X_L^s},$$

where \bar{P}_L denotes the average regional labor wage, \mathbf{z}_L^s denotes the factors explaining heterogeneity of the quality of family labor regarding off-farm work, and $b^s(X_L^s, \mathbf{z}_L^s)$ denotes the upward or downward shift of the average labor wage observed by the farm household. We expect that b^s is non-increasing in labor supply, according to our expositions above.

Taking observable heterogeneity and transaction costs into account, the effective revenues from off-farm employment are a function of supplied labor time:

$$(6) \quad f(X_L^s, \mathbf{z}_L^s, \mathbf{z}_v^s, \mathbf{z}_f^s) = \bar{P}_L X_L^s + \int_0^{X_L^s} b(\mathcal{X}, \mathbf{z}_L^s) d\mathcal{X} - TC_v^s(X_L^s, \mathbf{z}_v^s) - Y^s TC_f^s(\mathbf{z}_f^s),$$

where Y^s equals one if $X_L^s > 0$, and zero otherwise; $TC_f^s(\mathbf{z}_f^s)$ denotes fixed transaction costs, and \mathbf{z}_f^s are factors explaining fixed transaction costs of supplying off-farm labor.

Moreover, observed heterogeneity of on-farm labor might also affect labor demand. For example, some studies (Benjamin 1992; Deolalikar and Vijverberg 1983, 1987; Frisvold 1994) point out that different productivity might be observed for hired and family farm labor. We assume that farm-specific productivity also varies across hired workers. As long as this is unobservable by the farm household, heterogeneity implies transaction costs. However, even if farm households observe farm-specific labor productivity of various workers, it still might affect farmers' price responses if it is not fully reflected in the wage rate. Assuming a constant market wage rate for labor, it is rational to hire workers in the order that corresponds to their on-farm productivity. Under this assumption the marginal cost of an effective unit of on-farm labor is a step-wise increasing function of hired on-farm labor. Again, we approximate this step-wise

labor cost function by a continuous function and add marginal transaction costs to obtain the effective marginal wage rate:

$$(7) \quad P_L^h = \bar{P}_L + b^h(X_L^h, \mathbf{z}_L^h) + \frac{\partial TC_v^h(X_L^h, \mathbf{z}_v^h)}{\partial X_L^h},$$

where \mathbf{z}_L^h denotes the factors explaining heterogeneity of hired on-farm labor, and $b^h(X_L^h, \mathbf{z}_L^h)$ denotes the upward or downward shift of the average regional labor wage observed by the farm household. Again, according to our above expositions, we expect that b^h is non-decreasing in labor demand. Taking into account heterogeneity and variable transaction costs on the labor demand side, the effective labor costs result as a function of demanded labor time:

$$(8) \quad g(X_L^h, \mathbf{z}_L^h, \mathbf{z}_v^h, \mathbf{z}_f^h) = \bar{P}_L X_L^h + \int_0^{X_L^h} b^h(\mathcal{X}, \mathbf{z}_L^h) d\mathcal{X} + TC_v^h(X_L^h, \mathbf{z}_v^h) + Y^h TC_f^h(\mathbf{z}_f^h),$$

where Y^h equals one, if $X_L^h > 0$ and zero otherwise, $TC_f^h(\mathbf{z}_f^h)$ denotes fixed transaction costs, and \mathbf{z}_f^h are factors explaining fixed transaction costs of demanding on-farm labor.

The higher the NTC or heterogeneity, the higher the decrease in the decision price of off-farm labor induced by increasing labor supply and the higher the increase in the decision price of hired on-farm labor induced by an increasing labor demand. Since it holds $\partial P_L^s / \partial X_L^s = \partial^2 f / \partial X_L^{s2}$ and $\partial P_L^h / \partial X_L^h = \partial^2 g / \partial X_L^{h2}$, the degree of this market imperfection can be measured by the second-order differentials of f and g . With no heterogeneity and no NTC, both functions are linear and both second-order differentials become zero. Hence, in this case, once households participate in labor markets, marginal off-farm income or marginal costs for hired labor are equal to the exogenously given regional wage rate (\bar{P}_L), corrected for proportional transaction costs as well as for household-specific wage shifters. Thus, if households participate in one of the labor markets, the farm household model becomes separable and delivers standard microeconomic comparative static results (Sadoulet, de Janvry, and Benjamin 1998). Of course, if fixed or proportional transaction costs are too high, households may still abstain from the labor market and stay autarkic, implying a non-separable FHM (Key, Sadoulet, and de Janvry 2000).

In contrast, when labor markets are imperfectly competitive due to heterogeneity or NTC, both functions are non-linear. In this case, the shadow price of labor (P_L^*) is endogenously determined and production and consumption decisions are determined by solving the utility maximization problem (1) to (4). Hence, non-separability occurs, even when households participate in labor markets. However, although non-linearity of the f or g function clearly indicates labor market imperfection due to heterogeneity or NTC or both, it is generally impossible to separate the partial impacts of NTC or heterogeneity from observed curvature properties (second-order differentials) of the f and g functions alone.

Theoretically, the curvature properties of the labor revenue function f and the labor cost function g are ambiguous. However, for analytical convenience, we assume f to be concave

and g to be convex, since a non-concave labor revenue or a non-convex cost function makes the FHM approach less tractable. Since FTC create discontinuities in the f and g functions, solutions to the maximization problem (1) to (4) cannot be found by simply solving the first-order conditions. Thus, we follow [Key, Sadoulet, and de Janvry \(2000\)](#) and decompose the solution in two steps. First, we solve for the optimal solution conditional on the labor market participation regime, and then choose the regime that leads to the highest utility. Assuming an interior solution for a given labor market regime (Y^h and Y^s), the optimal quantities of consumption and production goods and the allocation of time are determined by conditions (2) to (4) and the following equations with $\lambda, \phi, \mu > 0$; $C_m, C_a, C_L, X_c, X_a > 0$; $X_L, X_v < 0$; $X_L^s > 0$ if $Y^s = 1$ and $X_L^s = 0$ otherwise; $X_L^h > 0$ if $Y^h = 1$ and $X_L^h = 0$ otherwise.

$$(9) \quad \frac{\partial U(\cdot)}{\partial C_i} - \lambda P_i^{(*)} = 0 \quad i \in \{m, a, L\}$$

$$(10) \quad \phi \frac{\partial G(\cdot)}{\partial X_i} + \lambda P_i^{(*)} = 0 \quad i \in \{c, a, v, L\}$$

$$(11) \quad \frac{\partial f(\cdot)}{\partial X_L^s} - P_L^* = 0 \quad \text{if } Y^s = 1$$

$$(12) \quad \frac{\partial g(\cdot)}{\partial X_L^h} - P_L^* = 0 \quad \text{if } Y^h = 1$$

where λ, ϕ are Lagrangian multipliers associated with the budget and the technology constraints, respectively. $P_L^* = \mu/\lambda$ denotes the unobservable shadow wage in the case of non-separability, where μ is the Lagrangian multiplier associated with the time constraint. In the separable version, P_L^* corresponds to the exogenous wage rate corrected for PTC and individual wage shifters.

Comparative Statics

In general, comparative statics are derived from the first-order conditions (2) to (4) and (9) to (12) and thus differ for each labor market regime. However, for simplicity we assume that the farm household simultaneously supplies off-farm labor and demands on-farm labor. Following the standard FHM literature ([de Janvry, Fafchamps, and Sadoulet 1991](#)), comparative statics of a non-separable FHM can be decomposed into the following two components:³

$$(13) \quad \frac{dQ}{dP_j} = \left. \frac{\partial Q}{\partial P_j} \right|_{P_L^* = \text{const.}} + \frac{\partial Q}{\partial P_L^*} \frac{dP_L^*}{dP_j}; \quad j \in \{c, a, v, m\}; \quad Q \in \{X_{c,a,v,L}, C_{m,a,L}, X_L^{s,h}\}$$

The first term on the right (direct component) represents the supply or demand reactions to changes in the exogenous prices, assuming a constant labor price (P_L^*). The second term

³ Since derivation of the comparative statics of an FHM is quite standard, we omit a detailed derivation here and only present the main equations. For a detailed derivation see for example [Strauss \(1986\)](#), [de Janvry, Fafchamps, and Sadoulet \(1991\)](#), or [Henning \(1994\)](#).

(indirect component) represents the adjustments to the changes in the shadow wage rate caused by changes in the same exogenous price.

Assuming separability, farm household's production and consumption adjustments coincide with the direct component of equation (13). In this case, a household's net-labor supply is obtained by subtracting farm labor input ($|X_L|$) and leisure (C_L) from its total labor endowment (T_L).

To determine the indirect component of the non-separable model, we derive the shadow price adjustment by applying the implicit function theorem to the time constraint (3) (de Janvry, Fafchamps, and Sadoulet 1991):

$$(14) \quad \frac{dP_L^*}{dP_j} = - \frac{\frac{\partial X_L}{\partial P_j} - \frac{\partial C_L}{\partial P_j}}{\frac{\partial X_L}{\partial P_L^*} + \frac{\partial X_L^h}{\partial P_L^*} - \frac{\partial X_L^s}{\partial P_L^*} - \frac{\partial C_L^H}{\partial P_L^*}}$$

The numerator on the right represents the change in time allocation due to increasing exogenous prices. The denominator of equation (14) indicates the change in time allocation caused by changes of the shadow wage rate. Equation (14) differs from a corresponding standard non-separable FHM assuming absent labor markets by the term $\Lambda = \partial X_L^h / \partial P_L^* - \partial X_L^s / \partial P_L^*$ in the denominator. This term measures the degree of labor market imperfection due to NTC or heterogeneity. Λ is implicitly determined by the first-order conditions (11) and (12), whereby: $\partial X_L^s / \partial P_L^* = \left(\partial^2 f / \partial X_L^{s2} \right)^{-1}$ and $\partial X_L^h / \partial P_L^* = \left(\partial^2 g / \partial X_L^{h2} \right)^{-1}$. Λ is always positive if f is concave and g is convex. As indicated earlier, the degree of labor market imperfection increases with the second-order differentials, $\partial^2 f / \partial X_L^{s2}$ and $\partial^2 g / \partial X_L^{h2}$, measured in absolute terms. In the extreme case of infinitely high NTC and labor heterogeneity, Λ approaches zero; hence, comparative statics of the model in (13) approximate the comparative statics derived from an autarkic labor market regime. In the opposite extreme case of zero NTC and perfect labor homogeneity, f and g are linear functions and Λ becomes infinity, implying that the induced shadow wage adjustment (14) is zero. Thus, the comparative statics of the model (13) would be approximating those of a separable FHM.

We derive the complete comparative statics for all exogenous prices based on equations (13) and (14) (see Henning and Henningsen 2007). It directly follows from our theoretical analysis above that generally comparative static effects differ across labor market regimes, where the differences between the non-separable and the separable FHM increase with the level of market imperfection due to NTC and heterogeneity.

Empirical Specification

We fully specify a non-separable farm household model that can be econometrically estimated to assess the question if and to what extent market imperfection influences price responses

of farm households. Although our FHM approach includes FTC, PTC, and NTC, as well as labor heterogeneity, our empirical analysis focuses on market imperfection due to NTC and heterogeneity. The empirical specification and estimation strategy are presented in this section and the section thereafter. A more comprehensive derivation is given in [Henning and Henningsen \(2007\)](#).

Production Technology

The production technology (2) is represented by a multi-input multi-output profit function from the symmetric normalized quadratic (SNQ) form ([Diewert and Wales 1987, 1992; Kohli 1993](#)). The corresponding netput equations of the four netputs specified in the theoretical model are given by:

$$(15) \quad X_{in}(\mathbf{p}_{pn}, \mathbf{r}_n) = \alpha_i + w_n^{-1} \sum_{j \in \{c, a, v, L\}} \beta_{ij} P_{jn} - \frac{1}{2} \theta_i w_n^{-2} \sum_{j \in \{c, a, v, L\}} \sum_{k \in \{c, a, v, L\}} \beta_{jk} P_{jn} P_{kn} \\ + \sum_{j \in \{g, k\}} \delta_{ij} R_{jn} + \frac{1}{2} \theta_i \sum_{j \in \{g, k\}} \sum_{k \in \{g, k\}} \gamma_{jk} R_{jn} R_{kn} \quad \forall i \in \{c, a, v, L\}$$

where n indicates the observation (household), $\mathbf{p}_{pn} = (P_{an}, P_{cn}, P_{vn}, P_{Ln})$ indicates the netput prices, $\mathbf{r}_n = (R_{gn}, R_{kn})$ indicates quasi-fixed factors, $w_n = \sum_{i \in \{c, a, v, L\}} \theta_i P_{in}$ is a factor to normalize prices, $\theta_i = \sum_n P_{in} |X_{in}| / \sum_n \sum_{j \in \{c, a, v, L\}} P_{jn} |X_{jn}|$; $i \in \{c, a, v, L\}$ are predetermined weights of the individual netput prices, and α_i , β_{ij} , δ_{ij} , and γ_{ij} are the parameters to be estimated. To identify all β_{ij} , we impose the restrictions $\sum_{j \in \{c, a, v, L\}} \beta_{ij} \bar{P}_j = 0$; $i \in \{c, a, v, L\}$, where \bar{P}_j are the mean prices ([Diewert and Wales 1987](#), p. 54). Homogeneity in prices is automatically attained by the functional form and symmetry requires $\beta_{ij} = \beta_{ji} \forall i, j \in \{c, a, v, L\}$.

Consumption Decisions

The preferences of the farm households (1) and the corresponding consumption decisions are specified by an Almost Ideal Demand System (AIDS) ([Deaton and Muellbauer 1980](#)), i.e. expenditure shares of consumer goods result in:

$$(16) \quad W_{in} = \alpha_i + \sum_{j \in \{m, a, L\}} \gamma_{ij} \ln P_{jn} + \beta_i \ln \frac{Y_n}{\wp_n} \quad \forall i \in \{m, a, L\}$$

$$(17) \quad \text{with } \ln \wp_n = \alpha_0 + \sum_{i \in \{m, a, L\}} \alpha_i \ln P_{in} + \frac{1}{2} \sum_{i \in \{m, a, L\}} \sum_{j \in \{m, a, L\}} \gamma_{ij} \ln P_{in} \ln P_{jn}$$

where $W_{in} = P_{in} C_{in} / Y_n$; $i \in \{m, a, L\}$ are the expenditure shares, Y_n indicates full income, \wp_n is the translog consumer price index, P_{in} ; $i \in \{m, a, L\}$ indicates the consumer prices, and α_i , β_i , and γ_{ij} are the parameters to be estimated. Adding-up requires $\sum_{i \in \{m, a, L\}} \alpha_i = 1$, $\sum_{i \in \{m, a, L\}} \beta_i = 0$, $\sum_{i \in \{m, a, L\}} \gamma_{ij} = 0$, and homogeneity in prices requires $\sum_{j \in \{m, a, L\}} \gamma_{ij} = 0$, and symmetry requires $\gamma_{ij} = \gamma_{ji} \forall i, j \in \{m, a, L\}$.

Labor Market Decisions

To allow imperfect labor markets due to FTC, PTC, and NTC, as well as (observed) heterogeneity, we assume a quadratic form for the labor income function f in (6) and the labor cost function g in (8), which can be interpreted as second-order approximations of the true labor cost and income functions, respectively. According to our theoretical expositions above, assuming quadratic f and g functions implies that the shadow wage functions (5) and (7) are linear:

$$(18) \quad P_L^* = \beta_0^s + X_L^s \beta_1^s + \mathbf{z}^{s'} \boldsymbol{\beta}^s$$

$$(19) \quad P_L^* = \beta_0^h + X_L^h \beta_1^h + \mathbf{z}^{h'} \boldsymbol{\beta}^h$$

As in equation (5), the vector \mathbf{z}^s includes factors that explain variable transaction costs (PTC and NTC) of supplying labor (\mathbf{z}_v^s) and the average skill level of a farm household (\mathbf{z}_L^s) as well as a proxy for the average regional wage level (\tilde{P}_L). Analogously, as in equation (7), the vector \mathbf{z}^h includes factors explaining PTC and NTC of hiring labor (\mathbf{z}_v^h) and the average skill of hired on-farm labor (\mathbf{z}_L^h) as well as a proxy for the average regional wage level (\tilde{P}_L). Moreover, since the quadratic functions are second-order approximations of the true f and g functions, their (local) curvature properties are fully captured by the coefficients β_1^s and β_1^h , respectively. Accordingly, we can separately test for the significance of NTC and heterogeneity in off-farm and in on-farm labor markets with a t-test. The null hypotheses correspond to $H_0 : \beta_1^h = 0$ and $H_0 : \beta_1^s = 0$.⁴ Non-separability is implied if both null hypotheses are rejected. However, even if one of the null hypotheses cannot be rejected, non-separability can still occur if the farm household does not participate in the corresponding labor market owing to high fixed or proportional transaction costs.⁵

Estimation Strategy

The econometric estimation of the empirical model specified above (15–19) is not straightforward, since shadow prices of labor cannot be observed directly. Therefore, we use a two-stage

⁴ Our estimation strategy does not permit the estimation of FTC, since TC_f^s and TC_f^h cannot be identified. However, because we are only interested in the impact of imperfect labor markets on price responses, we do not need to identify fixed transaction costs at this stage and we let them be captured by exogenous transfers (E). The simultaneous estimation of FTC, PTC, and NTC is an interesting research topic (see [Vakis, Sadoulet, and de Janvry 2003](#)), which will require more elaboration in future work.

⁵ Non-linearity of the labor revenue and labor cost functions is a sufficient, but not a necessary condition for non-separability. It is, however, a necessary condition if households participate in labor markets. Even if the labor revenue and labor cost function are both linear, fixed or proportional transaction costs could be so high that farms abstain from labor markets and thus, their production and consumption decisions are no longer separable. Hence, if our statistical test rejects linearity of the labor revenue and labor cost functions, we can conclude that the FHM is generally non-separable. However, if our test does not reject linearity, we can conclude that the FHM is separable for households that participate in labor markets; nevertheless non-separability could still be observed in autarkic households. Other tests of separability have been suggested for the latter case (see for example [Benjamin 1992](#)). However, we did not apply these additional tests because in our specific empirical application our test was sufficient to identify non-separability (see section “Data and Empirical Results”).

estimation strategy. We estimate shadow prices of labor at the first stage and at the second stage we estimate separately the SNQ profit function (15), the Almost Ideal Demand System (16, 17) and the linear labor wage equations (18, 19).

Estimating Shadow Values of Labor (Stage 1)

We follow Lopez (1984) to estimate the shadow prices of labor and estimate a restricted profit function with labor as a quasi-fixed input. Assuming constant returns to labor, Lopez (1984) derived the shadow wages of the households as shadow price of labor on the farm. The netput quantities per unit of labor that correspond to an SNQ profit function are

$$(20) \quad \frac{X_{in}(\mathbf{p}_{pn}, \mathbf{r}_n, X_{Ln})}{X_{Ln}} = \alpha_i + w_n^{-1} \sum_{j \in \{c, a, v\}} \beta_{ij} P_{jn} - \frac{1}{2} \theta_i w_n^{-2} \sum_{j \in \{c, a, v\}} \sum_{k \in \{c, a, v\}} \beta_{jk} P_{jn} P_{kn} \\ + \sum_{j \in \{g, k\}} \delta_{ij} R_{jn} + \frac{1}{2} \theta_i \sum_{j \in \{g, k\}} \sum_{k \in \{g, k\}} \gamma_{jk} R_{jn} R_{kn} \quad \forall i \in \{c, a, v\}$$

where $\mathbf{p}_{pn} = (P_{an}, P_{cn}, P_{vn})$ indicates the netput prices. Parameters in equation (20) are analogously defined and we impose the analogous restrictions as in equation (15).

Finally, the shadow prices of labor can be obtained from the estimation results by

$$(21) \quad P_{Ln}^* = \frac{\partial \hat{\Pi}_n(\mathbf{p}_{pn}, \mathbf{r}_n, X_{Ln})}{\partial X_{Ln}}$$

where $\hat{\Pi}_n(\mathbf{p}_{pn}, \mathbf{r}_n, X_{Ln}) = \sum_{i \in \{c, a, v\}} P_{in} \hat{X}_{in}$ is the fitted variable profit of the n^{th} farm and $\hat{X}_{in}(\mathbf{p}_{pn}, \mathbf{r}_n, X_{Ln})$ are the fitted values of the netput quantities.

Microeconomic theory generally requires that profit functions are convex in all netput prices, which is not the case in many empirical estimations. Therefore, we impose convexity of the profit function (20), applying a three-step procedure suggested by Koebel, Falk, and Laisney (2003) based on the minimum distance and asymptotic least squares estimation (Gourieroux, Monfort, and Trognon 1985; Kodde, Palm, and Pfann 1990).⁶ First, the unrestricted (linear) netput equations are estimated to calculate the Hessian matrix of the unrestricted profit function. Second, we minimize the weighted difference between this unrestricted Hessian and a Hessian that is restricted to be positive semi-definite by the Cholesky factorization. Third, restricted coefficients are identified by an asymptotic least squares (ALS) framework. The weighting matrix for the minimization of the difference between the unrestricted and the restricted Hessian

⁶We first tried to impose convexity by a non-linear estimation using the Cholesky decomposition (Lau 1978b). However, the estimation of the restricted non-linear netput equations did not converge. The new procedure suggested by Koebel, Falk, and Laisney (2003) circumvents this non-linear estimation and is asymptotically equivalent to a (successful) non-linear estimation with convexity imposed. To retain convexity of the SNQ profit function, it would be sufficient to minimize the difference between the estimated (unrestricted) β coefficients and the (linearly independent) values of a restricted β coefficient matrix (Koebel 1998). However, this procedure adjusts only the β -coefficients, while the approach of Koebel, Falk, and Laisney (2003) adjusts *all* coefficients. Thus, the fit of the constrained model is much better, due to the flexibility of the other coefficients. Both approaches yield the same β s.

matrix is the inverse of the variance-covariance matrix of the Hessian matrix, which can be derived from the coefficient variance-covariance matrix of the unrestricted estimation. The variance covariance matrix of the coefficients is obtained by bootstrapping (Efron 1979; Efron and Tibshirani 1993).

Farm Technology (Stage 2a)

Given the estimated shadow prices of labor, we estimate the SNQ netput equations (15). Again, we impose convexity with the method of Koebel, Falk, and Laisney (2003). However, the price of labor (P_L^*) is endogenous and a generated regressor. We use a three-stage least squares (3SLS) estimation with the variables \mathbf{z} (see below) as instrumental variables for P_L^* , to account for the endogeneity and the generation of P_L^* (Pagan 1984) and to allow for contemporaneous correlation of the disturbance terms.

Consumption (Stage 2b)

Analogously, given the estimated shadow prices of leisure (labor), we estimate the demand system (16, 17). In addition to P_L^* being endogenous and a generated regressor, the full income variable (Y) in the consumption decision specification might be endogenous and depends on P_L^* . To avoid estimation biases, we utilize a three-stage least squares (3SLS) estimation, in which we use the variables \mathbf{z} (see below) as instruments for P_L^* and Y . To avoid non-linear estimation, the share equations of the AIDS are estimated by the “Iterated Linear Least Squares Estimator” (ILLE) proposed by Blundell and Robin (1999).

Labor Market Decisions (Stage 2c)

Given the estimated shadow prices of labor, we estimate the two linear labor wage functions (18) and (19). However, these estimations might be plagued by a sample selection bias and an endogeneity problem.⁷

The endogeneity problem arises because the regressors X_L^s and X_L^h are probably correlated with the disturbance terms. To overcome this problem, we use a 2SLS estimation and substitute fitted values ($\widehat{X}_L^s, \widehat{X}_L^h$) for the observed quantities of supplied and hired labor (X_L^s, X_L^h). According to our theory, the optimal labor market allocation (X_L^s, X_L^h) of households that supply and demand labor simultaneously depends on the first-order conditions (9)–(12). For households that only supply labor, the optimal amount of supplied labor (X_L^s) depends only on conditions (9)–(11), while for households that only demand labor, the optimal quantity of hired labor (X_L^h) depends only on conditions (9), (10), and (12). Therefore, the impact of exogenous variables on the

⁷ The deviations between the estimated and the unobserved (true) shadow prices of labor get a part of the regular error terms \mathbf{v}^s and \mathbf{v}^h of the shadow price equations (24) and (25). We assume that these deviations are neither correlated with the regressors \mathbf{z}^s and \mathbf{z}^h nor with the variables used as instruments for X_L^s and X_L^h in the 2SLS estimation ($\mathbf{z}_x^s, \mathbf{z}_x^s, \mathbf{z}_x^h$). Note that we do not have to assume that the deviations are uncorrelated with the regressors X_L^s and X_L^h because X_L^s and X_L^h are not used as instruments in the 2SLS estimation.

amount of traded labor (X_L^s, X_L^h) depends on the labor market regime. Hence, the first stage of this 2SLS estimation corresponds to a switching regression model.

The sample selection bias occurs because these equations can only be estimated for households that participate in labor markets. To correct for selectivity, we apply an extended Heckman procedure and add selectivity terms (λ) to these equations, which can be interpreted as an extension of the two-stage probit method for simultaneous equation models with selectivity suggested by [Lee, Maddala, and Trost \(1980\)](#). Assumptions about the error terms are given in [Henning and Henningsen \(2007\)](#). Overall, a consistent estimation of these functions corresponds to the joint estimation of the following eight equations:

Market participation equations (estimated as a bivariate probit model):

$$(22) \quad Y^{s*} = \mathbf{z}'\boldsymbol{\gamma}^s + \boldsymbol{\varepsilon}^s \quad \text{with } Y^{s*} > 0 \text{ if } X_L^s > 0 \text{ and } Y^{s*} \leq 0 \text{ if } X_L^s = 0$$

$$(23) \quad Y^{h*} = \mathbf{z}'\boldsymbol{\gamma}^h + \boldsymbol{\varepsilon}^h \quad \text{with } Y^{h*} > 0 \text{ if } X_L^h > 0 \text{ and } Y^{h*} \leq 0 \text{ if } X_L^h = 0$$

Shadow wage equations (second stage of the 2SLS estimation):

$$(24) \quad P_L^* = \beta_0^s + \widehat{X}_L^s \beta_1^s + \mathbf{z}'\boldsymbol{\beta}^s + \sigma^s \lambda^s + \mathbf{v}^s \quad \text{if } Y^{s*} > 0$$

$$(25) \quad P_L^* = \beta_0^h + \widehat{X}_L^h \beta_1^h + \mathbf{z}'\boldsymbol{\beta}^h + \sigma^h \lambda^h + \mathbf{v}^h \quad \text{if } Y^{h*} > 0$$

Labor supply and demand equations (first stage of the 2SLS estimation):

$$(26) \quad X_L^s = \mathbf{z}_x^{b'} \boldsymbol{\delta}_s^b + \sigma_s^{bs} \lambda^{bs} + \sigma_s^{bh} \lambda^{bh} + \xi_s^b \quad \text{if } Y^{s*} > 0 \wedge Y^{h*} > 0$$

$$(27) \quad X_L^s = \mathbf{z}_x^{s'} \boldsymbol{\delta}_s^s + \sigma_s^{ss} \lambda^{ss} + \sigma_s^{sh} \lambda^{sh} + \xi_s^s \quad \text{if } Y^{s*} > 0 \wedge Y^{h*} \leq 0$$

$$(28) \quad X_L^h = \mathbf{z}_x^{b'} \boldsymbol{\delta}_h^b + \sigma_h^{bs} \lambda^{bs} + \sigma_h^{bh} \lambda^{bh} + \xi_h^b \quad \text{if } Y^{h*} > 0 \wedge Y^{s*} > 0$$

$$(29) \quad X_L^h = \mathbf{z}_x^{h'} \boldsymbol{\delta}_h^h + \sigma_h^{hs} \lambda^{hs} + \sigma_h^{hh} \lambda^{hh} + \xi_h^h \quad \text{if } Y^{h*} > 0 \wedge Y^{s*} \leq 0$$

where $\mathbf{z} = (1, \mathbf{z}^{\pi'}, \mathbf{z}^{u'}, \mathbf{z}^{s'}, \mathbf{z}^{h'}, \mathbf{z}_f^{s'}, \mathbf{z}_f^{h'})'$ are factors influencing labor market participation; $\mathbf{z}_x^b = (1, \mathbf{z}^{\pi'}, \mathbf{z}^{u'}, \mathbf{z}^{s'}, \mathbf{z}^{h'})'$, $\mathbf{z}_x^s = (1, \mathbf{z}^{\pi'}, \mathbf{z}^{u'}, \mathbf{z}^{s'})'$, and $\mathbf{z}_x^h = (1, \mathbf{z}^{\pi'}, \mathbf{z}^{u'}, \mathbf{z}^{h'})'$ are factors influencing the quantity of supplied and hired labor (depending on the labor market regime); all $\boldsymbol{\varepsilon}$, \mathbf{v} , and $\boldsymbol{\xi}$ denote the error terms; all $\boldsymbol{\gamma}$, $\boldsymbol{\beta}$, $\boldsymbol{\sigma}$, and $\boldsymbol{\delta}$ are parameters or parameter vectors to be estimated, and the selectivity terms are

$$(30) \quad \lambda^s = \frac{\phi(\mathbf{z}'\boldsymbol{\gamma}^s)}{\Phi(\mathbf{z}'\boldsymbol{\gamma}^s)}, \quad \lambda^h = \frac{\phi(\mathbf{z}'\boldsymbol{\gamma}^h)}{\Phi(\mathbf{z}'\boldsymbol{\gamma}^h)}$$

$$(31) \quad \lambda^{bs} = \frac{\phi(\mathbf{z}'\boldsymbol{\gamma}^s) \Phi\left(\frac{\mathbf{z}'\boldsymbol{\gamma}^h - \rho \mathbf{z}'\boldsymbol{\gamma}^s}{\sqrt{1-\rho^2}}\right)}{\Phi_2(\mathbf{z}'\boldsymbol{\gamma}^s, \mathbf{z}'\boldsymbol{\gamma}^h)}, \quad \lambda^{bh} = \frac{\phi(\mathbf{z}'\boldsymbol{\gamma}^h) \Phi\left(\frac{\mathbf{z}'\boldsymbol{\gamma}^s - \rho \mathbf{z}'\boldsymbol{\gamma}^h}{\sqrt{1-\rho^2}}\right)}{\Phi_2(\mathbf{z}'\boldsymbol{\gamma}^s, \mathbf{z}'\boldsymbol{\gamma}^h)}$$

$$(32) \quad \lambda^{ss} = \frac{\phi(\mathbf{z}'\boldsymbol{\gamma}^s) \Phi\left(\frac{-\mathbf{z}'\boldsymbol{\gamma}^h + \rho \mathbf{z}'\boldsymbol{\gamma}^s}{\sqrt{1-\rho^2}}\right)}{\Phi_2^*(\mathbf{z}'\boldsymbol{\gamma}^s, -\mathbf{z}'\boldsymbol{\gamma}^h)}, \quad \lambda^{sh} = -\frac{\phi(\mathbf{z}'\boldsymbol{\gamma}^h) \Phi\left(\frac{\mathbf{z}'\boldsymbol{\gamma}^s - \rho \mathbf{z}'\boldsymbol{\gamma}^h}{\sqrt{1-\rho^2}}\right)}{\Phi_2^*(\mathbf{z}'\boldsymbol{\gamma}^s, -\mathbf{z}'\boldsymbol{\gamma}^h)}$$

$$(33) \quad \lambda^{hs} = -\frac{\phi(z'\gamma^s) \Phi\left(\frac{z'\gamma^h - \rho z'\gamma^s}{\sqrt{1-\rho^2}}\right)}{\Phi_2^*(-z'\gamma^s, z'\gamma^h)}, \quad \lambda^{hh} = \frac{\phi(z'\gamma^h) \Phi\left(\frac{-z'\gamma^s + \rho z'\gamma^h}{\sqrt{1-\rho^2}}\right)}{\Phi_2^*(-z'\gamma^s, z'\gamma^h)}$$

where $\phi()$ and $\Phi()$ denote the probability density (pdf) and cumulative distribution (cdf) function of a standard normal distribution, respectively, and Φ_2 and Φ_2^* are the cumulative distribution (cdf) functions of a bivariate standard normal distribution with correlations ρ and $-\rho$, respectively. A detailed derivation of the selectivity terms is available in [Henning and Henningsen \(2007\)](#).⁸ Equations (30) to (33) are used to compute the selectivity terms ($\hat{\lambda}$), which are then substituted for the true λ s in equations (24) to (29). The estimated results of equations (26) to (29) are then used to obtain fitted values (\hat{X}_L^s, \hat{X}_L^h) that are used to estimate the second stage of the 2SLS estimation of equations (24) and (25). Finally, the variance covariance matrix of the second stage coefficients are computed with the formula given in [Lee, Maddala, and Trost \(1980\)](#) to obtain consistent standard errors.

Data and Empirical Results

Data are based on an accounting survey of 202 agricultural households in several regions around Poznan (Mid-West Poland) in 1994. The data were collected by the Institute for Agriculture and Food Industries in Warsaw ([IERiGZ 1995](#)). Additional regional data are taken from [Główny Urząd Statystyczny \(1996\)](#) and [Zawadzki \(1994\)](#). Sample characteristics of different labor market regimes are presented in table 1.

The empirical specification of the theoretical model is as follows. On the production side, market goods (X_c) consist of all crop products, while animal products are considered as partly home-consumed goods (X_a). All relevant variable inputs of the farms are subsumed in netput X_v . Labor (X_L) includes both family (X_L^f) and hired labor (X_L^h). Land (R_g) and capital (R_k) are considered as quasi-fixed factors. On the consumption side, C_m includes all purchased consumption goods. The self-produced goods (C_a) correspond conceptually to the home-consumed animal products (X_a). The amount of leisure (C_L) is determined by calculating the yearly available time (T_L) of households minus on-farm (X_L^f) and off-farm (X_L^s) family labor.⁹

⁸ We thank Awudu Abdulai, who pointed out that [Saha, Love, and Schwart \(1994\)](#) analyze a similar sample selection problem. In particular, they suggest an extended Heckman procedure, which is also applied by [Abdulai, Monnin, and Gerber \(2005\)](#). Although we have been stimulated by their work, we actually derived slightly different selectivity terms. To compare our results with the results of [Saha, Love, and Schwart \(1994\)](#) we calculated the conditional expectation values by numerical integration and Monte Carlo simulation using the (free) statistical software R ([R Development Core Team 2005](#), see also <http://www.r-project.org>), and the add-on packages `adapt` ([Genz et al. 2005](#)), `mvtnorm` ([Genz, Bretz, and Hothorn 2005](#)), and `MASS` ([Venables and Ripley 2002](#)). While our formula perfectly fits the numerical calculations, the formula of [Saha, Love, and Schwart \(1994\)](#) did not.

⁹ It is assumed that each household member between 15 years and 60 years has 10 hours per day and each household member older than 60 years has five hours per day available for work and/or leisure. The annual available time of the household is calculated by multiplying the total hours per day of all household members by 365. We use the share of off-farm labor in total labor endowment (X_L^s/T_L) instead of the absolute amount

Table 1. Characteristics of the Different Labor Regimes

Variable	Unit	All	Sup. & Dem.	Only Sup.	Only Dem.	Autarkic
Number		199	57	47	61	34
N_k	number	1.3	1.5	1.3	1.4	0.7
N_w	number	2.8	2.8	3.2	2.4	3.0
N_o	number	0.7	0.6	0.6	0.8	0.7
A_h	years	43	41	44	43	45
T_L	hours	11399	11110	12891	10082	12185
$ X_L $	hours	3686	3579	3372	4040	3668
X_L^h	hours	211	278	0	430	0
X_L^s	hours	446	515	1266	0	0
X_L^n	hours	235	237	1266	-430	0
X_L^f	hours	3475	3301	3372	3610	3668
C_L	hours	7478	7295	8254	6473	8517
$P_m C_m$	1000 PLZ	91469	105939	78012	97792	74467
$P_a C_a$	1000 PLZ	19041	18487	19245	19939	18076
$P_c X_c$	1000 PLZ	132258	157581	65883	180020	95869
$P_a X_a$	1000 PLZ	212570	220643	123997	300046	164531
$P_v X_v $	1000 PLZ	211960	232143	117552	299629	151343
R_g	ha	14.7	16.9	9.4	18.3	11.7
R_k	1000 PLZ	649191	788881	425398	816534	424132
R_k/R_g	1000 PLZ / ha	46921	49666	48516	48134	37938
N_c	number	0.9	1.0	0.8	0.9	0.8
W_u	%	19	20	19	18	20
W_i	km/100 km ²	58	55	60	60	57
W_t	1/1000 popul.	48	47	49	49	47
W_r	%	45	44	50	43	46
\tilde{P}_L	Poland = 100	88	85	90	89	88
P_L^*	1000 PLZ/h	38	46	30	44	28

Note: Calculations are based on [IERiGZ \(1995\)](#). PLZ = Polish Zloty. Variables: N_k = number of family members up to 14 years, N_w = number of family members between 15 and 60 years, N_o = number of family members older than 60 years, A_h = age of the household head, T_L = total time available, $|X_L|$ = labor input on the farm, X_L^h = hired labor, X_L^s = supplied labor, X_L^n = net supplied labor, X_L^f = family labor input on the farm, C_L = leisure, $P_m C_m$ = value of consumed market goods, $P_a C_a$ = value of consumed self-produced goods, $P_c X_c$ = value of produced crop products, $P_a X_a$ = value of produced animal products, $P_v |X_v|$ = value of utilized variable inputs, R_g = amount of land of the farm, R_k = amount of capital of the farm, N_c = number of cars owned by the household, W_u = regional unemployment rate, W_i = regional density of the road and railroad network, W_t = regional density of telephones, W_r = proportion of the population that lives in rural areas, \tilde{P}_L = relative average regional wage level, P_L^* = endogenous shadow price of labor.

The variables \mathbf{z}^π influencing the shadow price of labor from the production side include land and capital endowments (R_g, R_k) as well as variable output and input prices (P_c, P_a, P_v). The variables \mathbf{z}^μ influencing the shadow price from the consumer side include household composition and consumer prices. In particular, household composition is measured by the number of family members up to 14 years (N_k), between 15 and 60 years (N_w), and older than 60 years (N_o), as well as sex (D_f), age (A_h), and age squared (A_h^2) of the household head, because these variables might influence the preferences for leisure.

The variable and fixed transaction costs on the labor markets ($\mathbf{z}_v^s, \mathbf{z}_v^h, \mathbf{z}_f^s, \mathbf{z}_f^h$) are explained by the number of cars owned by the household (N_c), the regional density of the road and railroad network (W_i), the regional number of telephones per 1,000 population (W_t), the regional unemployment rate (W_u), and the proportion of the population that lives in rural areas (W_r). Furthermore, we assume that the average off-farm skill level of farm households (\mathbf{z}_L^s) depends on the number of family members that are of working age (N_w), the number of family members older than 60 years (N_o), and the average level of human capital. Since no data on education is available, we follow [Vakis, Sadoulet, and de Janvry \(2003\)](#) and interpret sex (D_f), age (A_h), and age squared (A_h^2) of the household head as an indicator of average human capital. Finally, the average skill level of hired workers (\mathbf{z}_L^h) is explained by the mechanization on the farm, measured as capital intensity (R_k/R_g).

The sample contains two farms that do not produce any animal products, which are removed to provide a more homogeneous sample and to avoid imputing the unknown prices of animal products.

Estimation results

This section presents the main estimation results. More detailed results are available in [Henning and Henningsen \(2007\)](#). All estimations and calculations are carried out by the (free) statistical software “R” ([R Development Core Team 2005](#), see also <http://www.r-project.org>), using the add-on packages “micEcon” ([Henningsen and Toomet 2005](#)), “systemfit” ([Hamann and Henningsen 2006](#)), and “VGAM” ([Yee and Wild 1996](#)).

The three netput equations of the SNQ profit function (20) are estimated in the first step. The shadow prices of labor calculated from the restricted profit function have reasonable values for all but one farm household. This household has a negative shadow price and is therefore removed from the sample. Hence, the sample used includes 199 farm households.

Tables 2 and 3 present the estimates of the restricted second-step profit function (15) and of the Almost Ideal Demand System (16, 17), respectively.

of supplied labor (X_L^s) as an explanatory variable in the off-farm labor wage equation to account for different household sizes. Hence, we assume that the share of skilled and unskilled labor in the total household would not significantly vary with the family size. Using the absolute amount of off-farm labor supply instead does not change the main results, i.e. significant and negative impact on the effective off-farm wage rate.

Table 2. Estimation Results of the 2nd-Stage Profit Function

Parameter	$i = c$		$i = a$		$i = v$		$i = L$	
	Coef.	(t-val)	Coef.	(t-val)	Coef.	(t-val)	Coef.	(t-val)
α_i	-31261	(-2.31)	33699	(2.07)	-5480	(-0.37)	-62939	(-6.95)
β_{ic}	53083	(1.86)	64866	(2.75)	-84580	(-2.13)	-33368	(-3.46)
β_{ia}	64866	(2.75)	116773	(2.47)	-168328	(-2.68)	-13311	(-0.63)
β_{iv}	-84580	(-2.13)	-168328	(-2.68)	247344	(2.72)	5564	(0.32)
β_{iL}	-33368	(-3.46)	-13311	(-0.63)	5564	(0.32)	41115	(6.28)
δ_{ig}	6815	(4.59)	303	(0.14)	-6087	(-4.04)	-3181	(-2.81)
δ_{ik}	0.124	(4.40)	0.291	(7.49)	-0.167	(-6.97)	$7.87 \cdot 10^{-3}$	(0.20)
γ_{gg}	-172	(-1.28)						
γ_{gk}	$9.84 \cdot 10^{-3}$	(2.09)						
γ_{kk}	$-3.55 \cdot 10^{-7}$	(-2.26)						
R^2	0.747		0.492		0.821		0.278	

Note: For definitions of the estimated coefficients see equation (15), where the subscripts c , a , v , L , g , and k indicate crop products, animal products, variable inputs, labor, land, and capital, respectively. The standard errors of the coefficients are calculated using the bootstrap resampling method (Efron 1979; Efron and Tibshirani 1993). Monotonicity is fulfilled at 97.0% of the observations. The R^2 values are almost identical to the model without convexity imposed, indicating that the data do not unreasonably contradict the convexity constraint (see Henning and Henningsen 2007).

Table 3. Estimation Results of the AIDS

Parameter	$i = m$		$i = a$		$i = L$	
	Coef.	(t-val.)	Coef.	(t-val.)	Coef.	(t-val.)
α_i	0.555	(9.86)	0.185	(14.79)	0.260	(4.18)
β_i	-0.170	(-9.15)	-0.031	(-7.36)	0.201	(9.95)
γ_{im}	0.034	(1.28)	0.021	(0.79)	-0.055	(-5.34)
γ_{ia}	0.021	(0.79)	0.010	(0.35)	-0.031	(-9.36)
γ_{iL}	-0.055	(-5.34)	-0.031	(-9.36)	0.086	(7.97)
R^2	0.409		0.585		0.504	

Note: For definitions of the estimated coefficients see equation (16), where the subscripts m , a , and L indicate purchased market goods, self-produced goods, and leisure, respectively. The standard errors of the coefficients that have not been directly estimated are calculated with the formula of Klein (1953, p. 258). α_0 is set to 10.8, because this value gives the highest likelihood value of the AIDS Model. Monotonicity is fulfilled at 99.5% of the observations and concavity is fulfilled at 88.4% of the observations.

Table 4 presents the estimates of the off-farm and on-farm labor wage functions. Since the focus of this paper is on market imperfection due to NTC and heterogeneity, the parameters $\beta_1^s = \partial P_L^* / \partial X_L^s$ and $\beta_1^h = \partial P_L^* / \partial X_L^h$ are of particular interest. Recall that these coefficients measure the degree of market imperfection due to NTC and heterogeneity and thus are of particular relevance. The other coefficients, measuring the effects of the variables \mathbf{z} on labor market participation decisions and the influence of the variables \mathbf{z}^s and \mathbf{z}^h on the shadow prices of labor, are only of secondary interest and are explained in Henning and Henningsen (2007).¹⁰

As can be seen from table 4, the effect of labor supply on the off-farm wage rate (β_1^s) is significantly negative. This indicates a concave labor revenue function and, hence, increasing marginal NTC or heterogeneity of off-farm labor skills. If an average household increases off-farm labor supply by 1%, marginal revenue falls by 0.075%. The estimated parameter of the inverse Mill's ratio is not significantly different from zero, implying no sample selection bias.

¹⁰ It is somewhat disconcerting that many \mathbf{z} variables in table 4 do not have a statistically significant effect on labor market participation and effective wages. To some extent this is caused by multicollinearity of the regional variables. Although multicollinearity does not result in biased estimates, it reduces the precision of the estimated parameters of the correlated regressors, which leads to larger standard errors and thus, to less statistical significance. However, since we are predominantly interested in the effect of the traded quantities of labor on the effective wages, the lack of statistical significance of the \mathbf{z} variables has only minor negative consequences on the essential part of this article.

Table 4. Estimated Coefficients of Labor Market Equations

Regressor	Labor Supply		Labor Demand	
	1st Step: Probit	2nd Step: 2SLS	1st Step: Probit	2nd Step: 2SLS
Constant	-0.196	103.929 *	3.988	-31.012
X_L^s/T_L		-73.567 **		
X_L^h				0.047 ***
N_k	0.129		0.084	
N_w	0.158 *	-3.496 *	-0.382 ***	
N_o	-0.022	-3.945	-0.296 **	
D_f	0.388	-6.448	-0.199	
A_h	-0.003	2.190 *	-0.119 *	
A_h^2	$-7.1 \cdot 10^{-5}$	-0.025 *	$1.3 \cdot 10^{-3}$ *	
R_g	0.008		0.005	
R_k	$-6.5 \cdot 10^{-7}$		$1.9 \cdot 10^{-6}$ ***	
R_k/R_g	$1.0 \cdot 10^{-5}$		$-7.2 \cdot 10^{-6}$	$2.1 \cdot 10^{-4}$ **
P_c	3.091		3.836	
P_a	0.252		-0.115	
P_v	-1.608		-3.906	
N_c	0.142	-1.652	-0.139	4.511
W_u	-0.025	-0.427	-0.010	2.841 **
W_i	-0.030	-0.136	0.001	0.733 *
W_t	-0.007	-0.542	0.012	-0.217
W_r	0.034 **	-0.639 **	-0.030 *	-1.004 **
\tilde{P}_L	-0.014	-0.184	0.005	0.112
IMR Supply		-1.737		
IMR Demand				-15.987 **
ρ	-0.099		-0.099	
R^2		0.307		0.425

Note: *, **, and *** denote statistical significance at the 10%, 5% and 1% level, respectively. Wald test of the joint significance of the exclusion variables: labor supply $\chi^2 = 9.595$, $df = 7$, $p\text{-value} = 0.213$; labor demand $\chi^2 = 41.531$, $df = 11$, $p\text{-value} = 0.00002$. Variables: X_L^s = supplied labor [hours], T_L = total time available [hours], X_L^h = hired labor [hours], N_k = number of family members up to 14 years, N_w = number of family members between 15 and 60 years, N_o = number of family members older than 60 years, D_f = sex of the household head (male=0, female=1), A_h = age of the household head, R_g = amount of land of the farm [ha], R_k = amount of capital of the farm [1000 PLZ], P_c = price index of crop products (average=1), P_a = price index of animal products (average=1), P_v = price index of variable inputs (average=1), N_c = number of cars owned by the household, W_u = regional unemployment rate [%], W_i = regional density of the road and railroad network [km/100 km²], W_t = regional number of telephones per 1,000 population, W_r = proportion of the population that lives in rural areas [%], \tilde{P}_L = relative average regional wage level (Poland=100), IMR = inverse Mill's ratio.

The on-farm wage rate increases significantly with hired labor (table 4), indicating a convex labor cost function and thus the presence of increasing NTC or heterogeneity. Market imperfections appear more pronounced in on-farm labor markets than in off-farm labor markets. If an average household increases hired labor by 1%, the marginal cost rises by 0.259%. In contrast to the labor supply side, the estimated parameter of the inverse Mill's ratio is significantly different from zero.

We conclude that our estimated FHM is non-separable because the t-tests reject both null hypotheses.

Elasticities

Given our estimation results, we calculate the full set of price elasticities according to equations (13) and (14) using sample means. Elasticities for perfect labor markets (separable model) are computed using equation (13), setting the second term on the right (the indirect component) equal to zero. Elasticities for imperfect labor markets (non-separable model) are calculated for all four labor market regimes defined in the theoretical section. A detailed derivation of the FHM elasticities is available in [Henning and Henningsen \(2007\)](#).

To assess whether the degree of market imperfection has an impact on farm price responses, we compare the corresponding price elasticities across labor market regimes. The standard errors of the estimated price elasticities and the differences between elasticities derived for different labor market regimes are computed using the formula in [Klein \(1953, p. 258\)](#).

Table 5 summarizes the main results and shows the elasticities for three labor market regimes: perfect, imperfect, and missing labor markets. The reader is referred to [Henning and Henningsen \(2007\)](#) for a more comprehensive presentation of the elasticities and their standard errors.

Overall, we observe mixed results. For all consumer goods, crop products, and farm labor input, the degree of labor market imperfection has a significant influence on price responses. By contrast, price elasticities for animal products and variable inputs do not significantly differ across labor market regimes, indicating that the degree of market imperfection has only a negligible impact on household's price responses.

How can these results be explained? According to equation (13), for any good Q and any exogenous price P_j , $j \in \{c, a, v, m\}$, the difference in price elasticities between perfect and imperfect/missing labor market regimes equals $(\partial Q / \partial P_L^*) (P_L^* / Q) \cdot (dP_L^* / dP_j) (P_j / P_L^*)$. The first term denotes the cross-price elasticity for good Q with respect to the wage rate and the second term is the shadow price elasticity. The latter measures the impact of an exogenous price change on the shadow price of labor, while the first measures the change of the consumed or produced quantity of good Q induced by a change in the shadow price of labor.

Table 5. Estimated Price Elasticities of Farm Households

	P_c		P_a		P_v		P_m		P_L	
	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)
Perfect labor market (separable model)										
X_c	0.43 ^a	(1.99)	0.50 ^a	(2.90)	-0.57 ^a	(-2.03)	0.00 ^a		-0.36	(-3.77)
X_a	0.32 ^a	(2.90)	0.53 ^a	(2.49)	-0.73 ^a	(-2.62)	0.00 ^a		-0.12	(-0.88)
X_v	0.36 ^a	(2.03)	0.73 ^a	(2.62)	-1.08 ^a	(-2.69)	0.00 ^a		-0.00	(-0.01)
X_L	0.34 ^a	(3.77)	0.17 ^a	(0.88)	-0.00 ^a	(-0.01)	0.00 ^a		-0.51	(-6.29)
C_m	0.13 ^a	(6.08)	0.33 ^a	(3.26)	-0.21 ^a	(-6.08)	-0.67 ^a	(-6.80)	0.45	(4.20)
C_a	0.17 ^a	(7.70)	-0.55 ^a	(-1.25)	-0.27 ^a	(-7.70)	0.50 ^a	(1.20)	0.18	(0.41)
C_L	0.43 ^a	(42.25)	0.61 ^a	(39.18)	-0.69 ^a	(-42.25)	-0.19 ^a	(-9.46)	-0.07	(-3.22)
X_L^n	-19.15 ^a	(-13.18)	-22.20 ^a	(-7.11)	22.00 ^a	(9.08)	6.16 ^a	(9.46)	10.30	(7.07)
X_L^f	0.34 ^a	(3.77)	0.17 ^a	(0.88)	-0.00 ^a	(-0.01)	0.00 ^a		-0.51	(-6.29)
P_L^*	0.00 ^a		0.00 ^a		0.00 ^a		0.00 ^a		1.00	
Imperfect labor market (non-separable model: supplying and hiring labor)										
X_c	0.28 ^b	(1.53)	0.33 ^b	(2.09)	-0.40 ^b	(-1.55)	0.05 ^b	(2.56)		
X_a	0.27 ^a	(2.44)	0.48 ^a	(2.31)	-0.68 ^a	(-2.27)	0.02 ^a	(0.86)		
X_v	0.36 ^a	(2.10)	0.73 ^a	(2.57)	-1.08 ^a	(-2.62)	0.00 ^a	(0.01)		
X_L	0.14 ^b	(1.51)	-0.06 ^b	(-0.41)	0.23 ^b	(1.84)	0.07 ^b	(3.11)		
C_m	0.30 ^b	(5.87)	0.52 ^b	(4.64)	-0.40 ^b	(-6.42)	-0.72 ^b	(-7.28)		
C_a	0.22 ^b	(7.50)	-0.49 ^b	(-1.13)	-0.33 ^b	(-8.20)	0.49 ^b	(1.17)		
C_L	0.36 ^b	(15.23)	0.52 ^b	(17.96)	-0.60 ^b	(-20.81)	-0.17 ^b	(-7.99)		
X_L^n	-13.41 ^a	(-3.12)	-15.55 ^a	(-3.10)	15.41 ^a	(3.11)	4.31 ^a	(3.10)		
X_L^s	-5.47	(-3.46)	-6.34	(-3.28)	6.29	(3.38)	1.76	(3.28)		
X_L^h	1.30	(1.35)	1.51	(1.18)	-1.50	(-1.27)	-0.42	(-1.18)		
X_L^f	0.04 ^{ab}	(0.14)	-0.19 ^{ab}	(-0.55)	0.37 ^{ab}	(1.12)	0.11 ^{ab}	(1.17)		
P_L^*	0.40 ^b	(3.60)	0.46 ^b	(3.39)	-0.46 ^b	(-3.51)	-0.13 ^b	(-3.40)		
Missing labor market (non-separable model: autarkic in labor)										
X_c	-0.07 ^c	(-0.51)	-0.07 ^c	(-0.49)	0.00 ^c	(0.01)	0.16 ^c	(3.40)		
X_a	0.16 ^a	(0.88)	0.35 ^a	(1.32)	-0.55 ^a	(-1.44)	0.05 ^a	(0.92)		
X_v	0.35 ^a	(1.84)	0.72 ^a	(2.24)	-1.08 ^a	(-2.37)	0.00 ^a	(0.01)		
X_L	-0.35 ^c	(-5.77)	-0.63 ^c	(-9.15)	0.80 ^c	(11.88)	0.22 ^c	(6.00)		
C_m	0.69 ^c	(10.50)	0.97 ^c	(8.51)	-0.85 ^c	(-11.88)	-0.85 ^c	(-8.75)		
C_a	0.35 ^c	(5.16)	-0.34 ^c	(-0.82)	-0.48 ^c	(-6.07)	0.44 ^c	(1.08)		
C_L	0.17 ^c	(5.77)	0.31 ^c	(9.15)	-0.39 ^c	(-11.88)	-0.11 ^c	(-6.00)		
X_L^n	0.00 ^b		0.00 ^b		0.00 ^b		0.00 ^b			
X_L^f	-0.35 ^b	(-5.77)	-0.63 ^b	(-9.15)	0.80 ^b	(11.88)	0.22 ^b	(6.00)		
P_L^*	1.36 ^c	(9.17)	1.58 ^c	(9.44)	-1.56 ^c	(-9.79)	-0.44 ^c	(-5.65)		

Note: Variables: X = netput quantities, C = consumed quantities, P = exogenous prices, P^* = endogenous shadow prices; subscripts: c = crop products, a = animal products, v = variable inputs, L = labor/leisure; superscripts of X_L (labor quantities): f = family labor on the farm, h = hired, s = supplied, n = net supplied. For each specific elasticity the values that have a common alphabetic character do not differ significantly. For instance, the elasticity of X_c with respect to P_c has different letters for all three types of labor market imperfections, which means that these three values significantly differ. On the other hand, the elasticity of X_a with respect to P_a has the same letter for all three types of labor market imperfections, which means that these three values do not differ significantly.

Differences in price elasticities can thus result from either high cross-price elasticities or high shadow price elasticities, or both. Relatively high cross-price elasticities are observed for crop products (-0.36), farm labor input (-0.51) and purchased consumer goods (0.45) (see table 5). For these goods, we also observe the largest and statistically significant differences in price elasticities across market regimes. High shadow price elasticities were obtained for missing markets, while low values were found for imperfect labor markets. This reinforces our finding that the degree of imperfection due to NTC or heterogeneity is moderate. Among all commodity prices, the one for purchased consumer goods (P_m) has the lowest impact on the shadow price for labor, as can be seen from the right-hand column of table 5. This can be explained with reference to equation (14), where the numerator captures the commodity specific income and substitution effects. The lower these effects, the lower are the shadow price elasticities.

Table 5 also shows that adjustments of net labor supply ($X_L^n = X_L^s - X_L^h$) do not differ significantly between perfect and imperfect labor markets. However, for both regimes, these adjustments differ significantly from zero. Of course, labor adjustment is zero for missing markets.

Finally, in the Polish case, market imperfection reduces household's responses to exogenous price changes on the production side, i.e. most price elasticities decrease in absolute terms with the degree of market imperfection. For example, for perfect labor markets crop output and farm labor input show a clear positive response with respect to increased crop prices. These responses are significantly smaller if labor markets are imperfect, and become negative in missing labor markets, implying even an inverse supply response.

Conclusion

This article developed a farm household model that incorporates labor market imperfections due to fixed (FTC), proportional (PTC), and non-proportional variable (NTC) transaction costs as well as heterogeneity in on-farm and off-farm labor markets. In contrast to existing studies that incorporate only FTC and PTC, the model developed here allows for non-separability, even when households buy or sell labor.

Comparative static analysis indicates that price responses deviate from perfect labor markets, even when the household buys or sells labor, if NTC or labor heterogeneity exist. Furthermore, price elasticities in imperfect labor markets generally lie between the corresponding elasticities in absent and perfect labor markets.

The model also provides a quantitative measure of the degree of market imperfection due to NTC and heterogeneity, and allows for a test of whether NTC and heterogeneity can be excluded from the estimation without loss of explanatory power.

Applying the model to farm household data from Mid-West Poland shows that NTC and heterogeneity play a significant role in explaining households' behavior. However, in the Polish

case, market imperfection due to NTC or heterogeneity is rather moderate, with the effect of NTC and heterogeneity more pronounced when hiring on-farm labor than supplying off-farm labor. Econometric estimation of our generalized FHM approach is rather cumbersome, because we have to control simultaneously for various possible endogeneity and selectivity biases. Therefore, the question arises if this more complex model is worth the effort. From the perspective of policy makers, we must ask whether incorporating NTC and heterogeneity provides estimates of elasticities that are quite different from what could have been obtained otherwise. Here our analysis delivers mixed results. While differences are statistically significant and are considerable for all consumer and most producer goods, they are not for animal products and variable inputs.

Supplementary Appendix

Note: This supplementary appendix is published in the “AgEcon Search” online library (<http://agecon.lib.umn.edu>). In the main article this supplementary appendix is cited as “[Henning and Henningsen \(2007\)](#)”.

Motivation of the Labor Market Model

Simultaneous Demand of On-Farm Labor and Supply of Off-Farm Labor

Simultaneously demanding on-farm labor and supplying off-farm labor can be rational with a strictly convex labor cost function and a strictly concave labor income function. To observe this, assume that in autarky the shadow price of labor on the farm would be lower than the marginal revenue of selling off-farm labor and higher than the marginal cost of hiring on-farm labor. Obviously, under this assumption, utility maximizing implies that the farm household supplies off-farm labor until marginal revenue equals the shadow price of labor, while the household demands on-farm labor until marginal cost equals the shadow price of labor or hired labor equals optimal labor input, i.e. the household no longer works on its own farm. Now, given strict convex and strict concave labor costs and income functions, there always exists an interior solution, i.e. the household simultaneously supplies and demands labor and works on its own farm. For instance, if the skills of the household members to work off-farm are very heterogeneous, it is rational to simultaneously supply high-priced labor of well-educated household members and hire cheap agricultural labor (see also [Sadoulet, de Janvry, and Benjamin 1996](#)).

Examples of Non-proportional Variable Transaction Costs

In this section we provide some intuitive examples of non-proportional variable transaction costs (NTC). It is well recognized in the literature that participation in rural labor markets is often plagued by adverse selection and moral hazard problems due to asymmetric information regarding the quality of the labor force ([Eswaran and Kotwal 1986](#); [Spence 1976](#)) and the effort of hired labor, respectively ([Frisvold 1994](#); [Sadoulet, de Janvry, and Benjamin 1998](#); [Eswaran and Kotwal 1986](#)). Generally, moral hazard and adverse selection problems might change non-proportionally with the quantity of traded goods, implying NTC for both on-farm labor demand and off-farm labor supply. Theoretically, it is unclear how these costs vary, i.e. if they are increasing, decreasing, or proportional to the amount of hired or supplied labor.

For example, in the case of moral hazard problems of hired on-farm labor, it is well recognized that employers cannot easily infer labor effort indirectly by observing final output, due to the stochastic and seasonal nature of agricultural production. Therefore, supervision costs rise to control for moral hazard problems ([Frisvold 1994](#); [Feder 1985](#)). Marginal costs to supervise hired labor may increase along with the units of hired labor due to an increase in the probability

of free-riding, the greater importance of coordinating work inputs, and the increased effort to control for social conflicts among employees.

Moreover, adverse selection problems due to asymmetric information on the quality of hired labor might lead to transaction costs in rural labor markets. These transaction costs might be partially reduced by adequate formal institutions (Spence 1976). However, in rural labor markets, adequate formal institutions that avoid adverse selection problems, e.g. formal education certificates, are often incompletely developed. In that case, a firm might use informal screening mechanisms to learn about the quality of workers, e.g. information from peer groups or rural organizations (Granovetter 1973; Sadoulet, de Janvry, and Benjamin 1998). Accessibility to peer groups or rural organizations varies, i.e. workers living in the neighborhood might have more access than those living in a more distant village. Thus, the potential to control for adverse selection problems increases when firms shift their demand from local to regional labor markets, implying increasing marginal NTC.

Moreover, even if information, search, and bargaining costs are considered as fixed costs, they occur for each labor contract. Therefore, from the perspective of the farm household, total costs, including all labor contracts, are no longer fixed costs but vary with the number of workers. Finally, other transaction costs might also vary with the number of labor contracts, e.g. there are only slight additional costs if one or two people travel to the city in the same car or family members who work for the same firm might reduce search and bargaining costs for succeeding family workers. On the other hand, some part-time jobs might be available near the farm, while full-time jobs are only available in larger settlements farther away, implying increasing transportation costs.

Theoretical Results

Table A1. Theoretical Effects of Exogenous Price Changes

Behavior	Variable	Non-separable Model				Separable Model				
		P_c	P_a	P_v	P_m	P_c	P_a	P_v	P_L	P_m
Farm	X_c	?	?	?	?	+	?	(-)	(-)	0
	X_a	?	?	?	?	?	+	(-)	(-)	0
	$ X_v $?	?	?	?	(+)	(+)	-	(-)	0
	$ X_L $?	?	?	?	(+)	(+)	(-)	-	0
Consumption	C_m	(+)	(+)	(-)	?	(+)	(+)	(-)	(+)	(-)
	C_a	(+)	?	(-)	?	(+)	?	(-)	(+)	?
	C_L	?	?	?	?	(+)	(+)	(-)	?	?
Labor market	X_L^n	(-)	(-)	(+)	?	(-)	(-)	(+)	(+)	?
	X_L^s	(-)	(-)	(+)	?					
	X_L^h	(+)	(+)	(-)	?					
	P_L^*	(+)	(+)	(-)	?					

Note: It is assumed that goods are not inferior, technologies are not regressive, and households are net suppliers of labor and self-produced agricultural goods.

Variables: X = netput quantities, C = consumed quantities, P = exogenous prices, P^* = endogenous shadow prices; subscripts: c = crop products, a = animal products, v = variable inputs, L = labor/leisure; superscripts of X_L (labor quantities): h = hired, s = supplied, n = net supplied.

Symbols indication the direction of the effects:

- 0 = clear, no effect;
- +/- = clear, increase/decrease;
- (+)/(-) = unclear, but most likely an increase/decrease (assuming labor and variable inputs are complements, and consumption goods are net-substitutes);
- ? = unclear.

Symmetric Normalized Quadratic (SNQ) Profit Function

This functional form is also traded under the name of “symmetric generalized McFadden function” (Diewert and Wales 1992).

First Stage Profit Function

We follow [Lopez \(1984\)](#) and determine the shadow price of labor on the farm by estimating a profit function assuming constant returns to labor. In this case a symmetric normalized quadratic (SNQ) profit function ([Diewert and Wales 1987, 1992; Kohli 1993](#)) has following form:

$$(A1) \quad \Pi(\mathbf{p}_{pn}, \mathbf{r}_n, X_{Ln}) = X_{Ln} \left(\begin{aligned} & \sum_{i \in \{c, a, v\}} \alpha_i P_{in} + \frac{1}{2} w_n^{-1} \sum_{i \in \{c, a, v\}} \sum_{j \in \{c, a, v\}} \beta_{ij} P_{in} P_{jn} \\ & + \sum_{i \in \{c, a, v\}} \sum_{j \in \{g, k\}} \delta_{ij} P_{in} R_{jn} + \frac{1}{2} w_n \sum_{i \in \{g, k\}} \sum_{j \in \{g, k\}} \gamma_{ij} R_{in} R_{jn} \end{aligned} \right)$$

where n indicates the observation (household), Π is the profit function, X_{Ln} is the labor deployed on the farm, $w_n = \sum_{i \in \{c, a, v\}} \theta_i P_{in}$ is a factor to normalize prices, $\theta_i = \sum_n P_{in} |X_{in}| / \sum_n \sum_{j \in \{c, a, v\}} P_{jn} |X_{jn}|$; $i \in \{c, a, v\}$ are predetermined weights of the individual netput prices, $\mathbf{p}_{pn} = (P_{an}, P_{cn}, P_{vn})$ indicates the netput prices, X_{in} ; $i \in \{c, a, v\}$ denotes the quantity indices of the netputs, $\mathbf{r}_n = (R_{gn}, R_{kn})$ represents the quasi-fixed factors land (R_g) and capital (R_k), and α_i , β_{ij} , δ_{ij} , and γ_{ij} are the parameters to be estimated. To identify all β_{ij} , we impose the restrictions $\sum_{j \in \{c, a, v\}} \beta_{ij} \bar{P}_j = 0$; $i \in \{c, a, v\}$, where \bar{P}_j are the mean prices ([Diewert and Wales 1987](#), p. 54).

The corresponding netput equations can be obtained using Hotelling's Lemma:

$$(A2) \quad X_{in}(\mathbf{p}_{pn}, \mathbf{r}_n, X_{Ln}) = \frac{\partial \Pi(\mathbf{p}_{pn}, \mathbf{r}_n, X_{Ln})}{\partial P_{in}}$$

$$(A3) \quad = X_{Ln} \left(\begin{aligned} & \alpha_i + w_n^{-1} \sum_{j \in \{c, a, v\}} \beta_{ij} P_{jn} \\ & - \frac{1}{2} \theta_i w_n^{-2} \sum_{j \in \{c, a, v\}} \sum_{k \in \{c, a, v\}} \beta_{jk} P_{jn} P_{kn} \\ & + \sum_{j \in \{g, k\}} \delta_{ij} R_{jn} + \frac{1}{2} \theta_i \sum_{j \in \{g, k\}} \sum_{k \in \{g, k\}} \gamma_{jk} R_{jn} R_{kn} \end{aligned} \right)$$

Second Stage Profit Function

At the second stage we estimate a symmetric normalized quadratic (SNQ) profit function ([Diewert and Wales 1987, 1992; Kohli 1993](#)) with labor as variable input:

$$(A4) \quad \Pi(\mathbf{p}_{pn}, \mathbf{r}_n) = \sum_{i \in \{c, a, v, L\}} \alpha_i P_{in} + \frac{1}{2} w_n^{-1} \sum_{i \in \{c, a, v, L\}} \sum_{j \in \{c, a, v, L\}} \beta_{ij} P_{in} P_{jn} \\ + \sum_{i \in \{c, a, v, L\}} \sum_{j \in \{g, k\}} \delta_{ij} P_{in} R_{jn} + \frac{1}{2} w_n \sum_{i \in \{g, k\}} \sum_{j \in \{g, k\}} \gamma_{ij} R_{in} R_{jn}$$

where $w_n = \sum_{i \in \{c, a, v, L\}} \theta_i P_{in}$ is a factor to normalize prices, $\theta_i = \sum_n P_{in} |X_{in}| / \sum_n \sum_{j \in \{c, a, v, L\}} P_{jn} |X_{jn}|$; $i \in \{c, a, v, L\}$ are predetermined weights of the individual netput prices, $\mathbf{p}_{pn} = (P_{an}, P_{cn}, P_{vn}, P_{Ln})$ indicates the netput prices, X_{in} ; $i \in \{c, a, v, L\}$ denotes the quan-

tity indices of the netputs, $\mathbf{r}_n = (R_{gn}, R_{kn})$ represents the quasi-fixed factors land (R_g) and capital (R_k), and α_i , β_{ij} , δ_{ij} , and γ_{ij} are the parameters to be estimated. To identify all β_{ij} , we impose the restrictions $\sum_{j \in \{c, a, v, L\}} \beta_{ij} \bar{P}_j = 0$; $i \in \{c, a, v, L\}$, where \bar{P}_j are the mean prices (Diewert and Wales 1987, p. 54).

The corresponding netput equations can be obtained using Hotelling's Lemma:

$$(A5) \quad X_{in}(\mathbf{p}_{pn}, \mathbf{r}_n) = \frac{\partial \Pi(\mathbf{p}_{pn}, \mathbf{r}_n)}{\partial P_{in}}$$

$$(A6) \quad = \alpha_i + w_n^{-1} \sum_{j \in \{c, a, v, L\}} \beta_{ij} P_{jn} - \frac{1}{2} \theta_i w_n^{-2} \sum_{j \in \{c, a, v, L\}} \sum_{k \in \{c, a, v, L\}} \beta_{jk} P_{jn} P_{kn} \\ + \sum_{j \in \{g, k\}} \delta_{ij} R_{jn} + \frac{1}{2} \theta_i \sum_{j \in \{g, k\}} \sum_{k \in \{g, k\}} \gamma_{jk} R_{jn} R_{kn}$$

Labor Market Analysis

Labor Supply

To estimate the marginal revenue of supplying labor, we assume the following specifications of the average regional wage level \bar{P}_L , the household-specific wage shifters b^s , and the variable transaction costs TC_v^s , which include proportional (PTC) and non-proportional variable (NTC) transaction costs:

$$(A7) \quad \bar{P}_L = \tilde{P}_L \beta_p^s$$

$$(A8) \quad b^s(X_L^s, \mathbf{z}_L^s) = \beta_0^s + \mathbf{z}_L^{s'} \boldsymbol{\beta}_L^s + X_L^s \beta_{L1}^s$$

$$(A9) \quad TC_v^s(X_L^s, \mathbf{z}_v^s) = (\mathbf{z}_v^{s'} \boldsymbol{\beta}_v^s) X_L^s + \beta_{v1}^s X_L^{s2}$$

where \tilde{P}_L is a proxy for the average regional wage level. The specification of b^s shows that $\mathbf{z}_L^{s'} \boldsymbol{\beta}_L^s$ indicates general wage differences between the households, while $X_L^s \beta_{L1}^s$ refers to a wage shift due to a changing amount of supplied labor, which is caused by heterogeneity within each household. The specification of TC_v^s is derived from a second-order Taylor series approximation of the true transaction costs (see section). It shows that $\mathbf{z}_v^{s'} \boldsymbol{\beta}_v^s$ denotes proportional transaction costs per unit of labor, and $\beta_{v1}^s X_L^{s2}$ are non-proportional variable transaction costs.

Substituting these specifications into equation (5) of the main article, we get the empirical specification used for the estimation, which is presented in equation (18) of the main article.

$$(A10) \quad P_L^s = \bar{P}_L + b^s(X_L^s, \mathbf{z}_L^s) - \frac{\partial TC_v^s(X_L^s, \mathbf{z}_v^s)}{\partial X_L^s}$$

$$(A11) \quad = \tilde{P}_L \beta_p^s + \beta_0^s + \mathbf{z}_L^{s'} \boldsymbol{\beta}_L^s + X_L^s \beta_{L1}^s - \mathbf{z}_v^{s'} \boldsymbol{\beta}_v^s - 2X_L^s \beta_{v1}^s$$

$$(A12) \quad = \beta_0^s + \tilde{P}_L \beta_p^s + \mathbf{z}_L^{s'} \boldsymbol{\beta}_L^s - \mathbf{z}_v^{s'} \boldsymbol{\beta}_v^s + X_L^s (\beta_{L1}^s - 2\beta_{v1}^s)$$

$$(A13) \quad = \beta_0^s + \mathbf{z}^{s'} \boldsymbol{\beta}^s + X_L^s \beta_1^s$$

with $\mathbf{z}^s = (\tilde{P}_L, \mathbf{z}_L^{s'}, \mathbf{z}_v^{s'})'$, $\boldsymbol{\beta}^s = (\beta_p^s, \boldsymbol{\beta}_L^{s'}, -\boldsymbol{\beta}_v^{s'})'$, and $\beta_1^s = \beta_{L1}^s - 2\beta_{v1}^s$.

Neglecting FTC, we can derive the net off-farm labor revenue function f from the estimated coefficients of equation (A13) by applying equation (6) of the main article:

$$(A14) \quad f(X_L^s) + TC_f^s = \int_0^{X_L^s} \left(\widehat{\beta}_0^s + \mathbf{z}^{s'} \widehat{\boldsymbol{\beta}}^s + X_L^s \widehat{\beta}_1^s \right) dX_L^s = \left(\widehat{\beta}_0^s + \mathbf{z}^{s'} \widehat{\boldsymbol{\beta}}^s \right) X_L^s + \frac{1}{2} \widehat{\beta}_1^s X_L^{s2}$$

Labor Demand

To estimate the marginal cost of hiring labor, we assume the following specifications of the average regional wage level \bar{P}_L , the farm-specific wage shifters b^h , and the variable transaction costs TC_v^h , which include PTC and NTC:

$$(A15) \quad \bar{P}_L = \widetilde{P}_L \beta_p^h$$

$$(A16) \quad b^h \left(X_L^h, \mathbf{z}_L^h \right) = \beta_0^h + \mathbf{z}_L^{h'} \boldsymbol{\beta}_L^h + X_L^h \beta_{L1}^h$$

$$(A17) \quad TC_v^h \left(X_L^h, \mathbf{z}_v^h \right) = \left(\mathbf{z}_v^{h'} \boldsymbol{\beta}_v^h \right) X_L^h + \beta_{v1}^h X_L^{h2}$$

where \widetilde{P}_L is a proxy for the average regional wage level. The specification of b^h shows that $\mathbf{z}_L^{h'} \boldsymbol{\beta}_L^h$ indicates general wage differences between the farms, while $X_L^h \beta_{L1}^h$ refers to a wage shift due to a changing amount of hired labor, which is caused by heterogeneity within the hired workers of each farm. The specification of TC_v^h is derived from a second-order Taylor series approximation of the true transaction costs (see section). It shows that $\mathbf{z}_v^{h'} \boldsymbol{\beta}_v^h$ denotes proportional transaction costs per unit of labor, and $\beta_{v1}^h X_L^{h2}$ are non-proportional transaction costs.

Substituting these specifications into equation (7) of the main article, we get the empirical specification used for the estimation, which is presented in equation (19) of the main article.

$$(A18) \quad P_L^h = \bar{P}_L + b^h \left(X_L^h, \mathbf{z}_L^h \right) + \frac{\partial TC_v^h \left(X_L^h, \mathbf{z}_v^h \right)}{\partial X_L^h}$$

$$(A19) \quad = \widetilde{P}_L \beta_p^h + \beta_0^h + \mathbf{z}_L^{h'} \boldsymbol{\beta}_L^h + X_L^h \beta_{L1}^h + \mathbf{z}_v^{h'} \boldsymbol{\beta}_v^h + 2X_L^h \beta_{v1}^h$$

$$(A20) \quad = \beta_0^h + \widetilde{P}_L \beta_p^h + \mathbf{z}_L^{h'} \boldsymbol{\beta}_L^h + \mathbf{z}_v^{h'} \boldsymbol{\beta}_v^h + X_L^h \left(\beta_{L1}^h + 2\beta_{v1}^h \right)$$

$$(A21) \quad = \beta_0^h + \mathbf{z}^{h'} \boldsymbol{\beta}^h + X_L^h \beta_1^h$$

with $\mathbf{z}^h = \left(\widetilde{P}_L, \mathbf{z}_L^{h'}, \mathbf{z}_v^{h'} \right)'$, $\boldsymbol{\beta}^h = \left(\beta_p^h, \boldsymbol{\beta}_L^h, \boldsymbol{\beta}_v^h \right)'$, and $\beta_1^h = \beta_{L1}^h + 2\beta_{v1}^h$.

Neglecting FTC, we can derive the effective cost function for hired labor g from the estimated coefficients of equation (A21) by applying equation (8) of the main article:

$$(A22) \quad g \left(X_L^h \right) - TC_f^h = \int_0^{X_L^h} \left(\widehat{\beta}_0^h + \mathbf{z}^{h'} \widehat{\boldsymbol{\beta}}^h + \widehat{\beta}_1^h X_L^h \right) dX_L^h = \left(\widehat{\beta}_0^h + \mathbf{z}^{h'} \widehat{\boldsymbol{\beta}}^h \right) X_L^h + \frac{1}{2} \widehat{\beta}_1^h X_L^{h2}$$

Second-order Taylor Series Approximation of Variable Transaction Costs

We assume that the transactions costs (TC) are a function of the traded quantity (X_L) and some further factors that influence variable transaction costs (\mathbf{z}_v)¹¹:

$$(A23) \quad TC = f \left(\begin{array}{c} X_L \\ \mathbf{z}_v \end{array} \right)$$

where X_L is a scalar that represents X_L^s or X_L^h and \mathbf{z}_v is a vector that represents \mathbf{z}_v^s or \mathbf{z}_v^h .

We approximate the true transaction costs at point $\left(\begin{array}{c} X_L^0 \\ \mathbf{z}_v^0 \end{array} \right)$ by a second-order Taylor series¹²:

$$(A24) \quad TC^* = f \left(\begin{array}{c} X_L^0 \\ \mathbf{z}_v^0 \end{array} \right) + \left(\begin{array}{c} X_L - X_L^0 \\ \mathbf{z}_v - \mathbf{z}_v^0 \end{array} \right)' \left(\begin{array}{c} \frac{\partial TC}{\partial X_L} \\ \frac{\partial TC}{\partial \mathbf{z}_v} \end{array} \right) \\ + \left(\begin{array}{c} X_L - X_L^0 \\ \mathbf{z}_v - \mathbf{z}_v^0 \end{array} \right)' \left(\begin{array}{cc} \frac{\partial^2 TC}{\partial X_L^2} & \frac{\partial^2 TC}{\partial X_L \partial \mathbf{z}_v} \\ \frac{\partial^2 TC}{\partial X_L \partial \mathbf{z}_v} & \frac{\partial^2 TC}{\partial \mathbf{z}_v^2} \end{array} \right) \left(\begin{array}{c} X_L - X_L^0 \\ \mathbf{z}_v - \mathbf{z}_v^0 \end{array} \right)$$

$$(A25) \quad = f \left(\begin{array}{c} X_L^0 \\ \mathbf{z}_v^0 \end{array} \right) + (X_L - X_L^0) \frac{\partial TC}{\partial X_L} + (\mathbf{z}_v - \mathbf{z}_v^0)' \frac{\partial TC}{\partial \mathbf{z}_v} + (X_L - X_L^0)^2 \frac{\partial^2 TC}{\partial X_L^2} \\ + 2(X_L - X_L^0) \frac{\partial^2 TC}{\partial X_L \partial \mathbf{z}_v} (\mathbf{z}_v - \mathbf{z}_v^0) + (\mathbf{z}_v - \mathbf{z}_v^0)' \frac{\partial^2 TC}{\partial \mathbf{z}_v^2} (\mathbf{z}_v - \mathbf{z}_v^0)$$

$$(A26) \quad = f \left(\begin{array}{c} X_L^0 \\ \mathbf{z}_v^0 \end{array} \right) + \frac{\partial TC}{\partial X_L} X_L - \frac{\partial TC}{\partial X_L} X_L^0 + (\mathbf{z}_v - \mathbf{z}_v^0)' \frac{\partial TC}{\partial \mathbf{z}_v} \\ + \frac{\partial^2 TC}{\partial X_L^2} X_L^2 - 2 \frac{\partial^2 TC}{\partial X_L^2} X_L X_L^0 + \frac{\partial^2 TC}{\partial X_L^2} X_L^{02} + 2 \frac{\partial^2 TC}{\partial X_L \partial \mathbf{z}_v} (\mathbf{z}_v - \mathbf{z}_v^0) X_L \\ - 2 \frac{\partial^2 TC}{\partial X_L \partial \mathbf{z}_v} (\mathbf{z}_v - \mathbf{z}_v^0) X_L^0 + (\mathbf{z}_v - \mathbf{z}_v^0)' \frac{\partial^2 TC}{\partial \mathbf{z}_v^2} (\mathbf{z}_v - \mathbf{z}_v^0)$$

All terms that do not vary with X_L are considered as fixed transaction costs:

$$(A27) \quad TC_f^* = f \left(\begin{array}{c} X_L^0 \\ \mathbf{z}_v^0 \end{array} \right) - \frac{\partial TC}{\partial X_L} X_L^0 + (\mathbf{z}_v - \mathbf{z}_v^0)' \frac{\partial TC}{\partial \mathbf{z}_v} + \frac{\partial^2 TC}{\partial X_L^2} X_L^{02} \\ - 2 \frac{\partial^2 TC}{\partial X_L \partial \mathbf{z}_v} (\mathbf{z}_v - \mathbf{z}_v^0) X_L^0 + (\mathbf{z}_v - \mathbf{z}_v^0)' \frac{\partial^2 TC}{\partial \mathbf{z}_v^2} (\mathbf{z}_v - \mathbf{z}_v^0)$$

¹¹In this section we ignore factors that influence fixed transaction costs because we are interested only in variable transaction costs here.

¹²All derivatives are evaluated at point $\left(\begin{array}{c} X_L^0 \\ \mathbf{z}_v^0 \end{array} \right)$ but in the following this is omitted for better readability.

Now we get for the variable transaction costs

$$(A28) \quad TC_v^* = TC^* - TC_f^*$$

$$(A29) \quad = \frac{\partial TC}{\partial X_L} X_L + \frac{\partial^2 TC}{\partial X_L^2} X_L^2 - 2 \frac{\partial^2 TC}{\partial X_L^2} X_L X_L^0 + 2 \frac{\partial^2 TC}{\partial X_L \partial \mathbf{z}_v} (\mathbf{z}_v - \mathbf{z}_v^0) X_L$$

$$(A30) \quad = \left(\frac{\partial TC}{\partial X_L} - 2 \frac{\partial^2 TC}{\partial X_L^2} X_L^0 - 2 \frac{\partial^2 TC}{\partial X_L \partial \mathbf{z}_v} \mathbf{z}_v^0 + 2 \frac{\partial^2 TC}{\partial X_L \partial \mathbf{z}_v} \mathbf{z}_v \right) X_L + \frac{\partial^2 TC}{\partial X_L^2} X_L^2$$

$$(A31) \quad = (\tilde{\mathbf{z}}_v' \boldsymbol{\beta}_v) X_L + \beta_{v1} X_L^2$$

with

$$(A32) \quad \tilde{\mathbf{z}}_v = \begin{pmatrix} 1 \\ \mathbf{z}_v \end{pmatrix}$$

$$(A33) \quad \boldsymbol{\beta}_v = \begin{pmatrix} \frac{\partial TC}{\partial X_L} - 2 \frac{\partial^2 TC}{\partial X_L^2} X_L^0 - 2 \frac{\partial^2 TC}{\partial X_L \partial \mathbf{z}_v} \mathbf{z}_v^0 \\ 2 \frac{\partial^2 TC}{\partial X_L \partial \mathbf{z}_v} \end{pmatrix}$$

$$(A34) \quad \beta_{v1} = \frac{\partial^2 TC}{\partial X_L^2}$$

Exclusion Variables

In a two-step Heckman estimation, the variables that are regressors in the first-step selection equation (say, \mathbf{x}_1) but are not regressors in the second-step regression equation (say, \mathbf{x}_2) are called “exclusion variables.” If there are no exclusion variables ($\mathbf{x}_1 \subseteq \mathbf{x}_2$), the sample correction term in the second step (say, λ) is likely to be highly correlated with the other regressors in \mathbf{x}_2 because λ is a (non-linear) function of a linear combination of the variables in \mathbf{x}_1 ($\lambda = \phi(\mathbf{x}_1' \boldsymbol{\gamma}) / \Phi(\mathbf{x}_1' \boldsymbol{\gamma})$, where $\boldsymbol{\gamma}$ are the coefficients of the selection equation and ϕ and Φ are probability density function (pdf) and the cumulative distribution function (cdf) of the standard normal distribution, respectively). Hence, the purpose of exclusion variables is to reduce the correlation among the regressors (multicollinearity) in the second-step estimation. Although high multicollinearity does not result in biased estimates, it leads to large standard errors, which means that the estimates are rather imprecise.

The exclusion variables for the equations explaining the shadow price of labor can be identified from table 4 in the main article. The exclusion variables for the marginal revenue of labor supply (equation (24) in the main article) are the number of kids (N_k), land and capital endowment of the farm (R_g, R_k); the capital intensity on the farm (R_k/R_g); and the prices of farm netputs (P_c, P_a, P_v). The exclusion variables for the marginal cost of labor demand (equation (25) in the main article) are the age pattern of the household (N_k, N_w, N_o); sex, age, and age squared of the head of the household (D_f, A_h, A_h^2); land and capital endowment of the farm (R_g, R_k); and the prices of farm netputs (P_c, P_a, P_v).

The exclusion variables for the equations explaining the quantity of supplied labor (equations (26) and (27) in the main article) are variables that are in \mathbf{z} but not in \mathbf{z}_x^b and \mathbf{z}_x^s , respectively. The exclusion variables for the equations explaining the quantity of hired labor (equations (28) and (29) in the main article) are variables that are in \mathbf{z} but not in \mathbf{z}_x^b and \mathbf{z}_x^h , respectively. Theoretically, the exclusion variables in (26) and (28) are the variables that are in \mathbf{z}_f^s or \mathbf{z}_f^h but not in \mathbf{z}^π , \mathbf{z}^u , \mathbf{z}^s , or \mathbf{z}^h , the exclusion variables in (27) are the variables that are in \mathbf{z}_f^s , \mathbf{z}_f^h or \mathbf{z}^h but not in \mathbf{z}^π , \mathbf{z}^u or \mathbf{z}^s , and the exclusion variables in (29) are the variables that are in \mathbf{z}_f^s , \mathbf{z}_f^h or \mathbf{z}^s but not in \mathbf{z}^π , \mathbf{z}^u or \mathbf{z}^h . However, in practice, our data set does not include any variables that influence fixed transaction costs (\mathbf{z}_f^s , \mathbf{z}_f^h) but do not influence variable transaction costs or the average skill level (\mathbf{z}^s , \mathbf{z}^h). Thus, given the specification of the \mathbf{z} variables in section “Data and Empirical Results” in the main article, we have an exclusion variable only in (27) (R_K/R_g) but not in the other three X equations. Although this leads to multicollinearity, it does not matter in our special case because we are interested in the fitted values but not the estimated coefficients. As long as multicollinearity is not so high that it rules out estimation, we can calculate fitted values that are orthogonal to the error terms of the estimations of the shadow price of labor (given that the regressors are not correlated with these error terms, too).

Assumptions about Error Terms

We assume that the residuals of the participation equations (22, 23) in the main article, $\boldsymbol{\varepsilon}^s$ and $\boldsymbol{\varepsilon}^h$, have a bivariate normal distribution:

$$(A35) \quad \begin{pmatrix} \boldsymbol{\varepsilon}^s \\ \boldsymbol{\varepsilon}^h \end{pmatrix} \sim N \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{bmatrix} 1 & \rho \\ & 1 \end{bmatrix} \right)$$

Further, we assume a joint normal distribution of $\boldsymbol{\varepsilon}^s$, $\boldsymbol{\varepsilon}^h$, $\tilde{\mathbf{v}}^s$ and $\tilde{\mathbf{v}}^h$ with covariances $\boldsymbol{\sigma}^s = \text{cov}(\tilde{\mathbf{v}}^s, \boldsymbol{\varepsilon}^s)$ and $\boldsymbol{\sigma}^h = \text{cov}(\tilde{\mathbf{v}}^h, \boldsymbol{\varepsilon}^h)$, where $\tilde{\mathbf{v}}^s$ and $\tilde{\mathbf{v}}^h$ would be the error terms of equations (24) and (25) in the main article, respectively, without selectivity terms. From this we can obtain the conditional expectation of the error terms

$$(A36) \quad E[\tilde{\mathbf{v}}^s | Y^{s*} > 0] = \boldsymbol{\sigma}^s \boldsymbol{\lambda}^s$$

$$(A37) \quad E[\tilde{\mathbf{v}}^h | Y^{h*} > 0] = \boldsymbol{\sigma}^h \boldsymbol{\lambda}^h$$

where $\boldsymbol{\lambda}^s$ and $\boldsymbol{\lambda}^h$ are defined as in equation (30) of the main article.

Furthermore, we assume a joint normal distribution of $\boldsymbol{\varepsilon}^s$, $\boldsymbol{\varepsilon}^h$, $\tilde{\boldsymbol{\xi}}_s^b$, $\tilde{\boldsymbol{\xi}}_s^s$, $\tilde{\boldsymbol{\xi}}_h^b$, and $\tilde{\boldsymbol{\xi}}_h^h$ with covariances $\boldsymbol{\sigma}_s^{bs} = \text{cov}(\tilde{\boldsymbol{\xi}}_s^b, \boldsymbol{\varepsilon}^s)$, $\boldsymbol{\sigma}_s^{bh} = \text{cov}(\tilde{\boldsymbol{\xi}}_s^b, \boldsymbol{\varepsilon}^h)$, $\boldsymbol{\sigma}_s^{ss} = \text{cov}(\tilde{\boldsymbol{\xi}}_s^s, \boldsymbol{\varepsilon}^s)$, $\boldsymbol{\sigma}_s^{sh} = \text{cov}(\tilde{\boldsymbol{\xi}}_s^s, \boldsymbol{\varepsilon}^h)$, $\boldsymbol{\sigma}_h^{bs} = \text{cov}(\tilde{\boldsymbol{\xi}}_h^b, \boldsymbol{\varepsilon}^s)$, $\boldsymbol{\sigma}_h^{bh} = \text{cov}(\tilde{\boldsymbol{\xi}}_h^b, \boldsymbol{\varepsilon}^h)$, $\boldsymbol{\sigma}_h^{hs} = \text{cov}(\tilde{\boldsymbol{\xi}}_h^s, \boldsymbol{\varepsilon}^s)$, and $\boldsymbol{\sigma}_h^{hh} = \text{cov}(\tilde{\boldsymbol{\xi}}_h^s, \boldsymbol{\varepsilon}^h)$, where $\tilde{\boldsymbol{\xi}}_s^b$, $\tilde{\boldsymbol{\xi}}_s^s$, $\tilde{\boldsymbol{\xi}}_h^b$, and $\tilde{\boldsymbol{\xi}}_h^h$ would be the error terms of equations (26), (27), (28), and (29) in the main article, respectively, without selectivity terms. From this we can obtain the conditional expectation of the error terms

$$(A38) \quad E[\tilde{\boldsymbol{\xi}}_s^b | Y^{s*} > 0 \wedge Y^{h*} > 0] = \boldsymbol{\sigma}_s^{bs} \boldsymbol{\lambda}^{bs} + \boldsymbol{\sigma}_s^{bh} \boldsymbol{\lambda}^{bh}$$

$$(A39) \quad E \left[\tilde{\xi}_s^s | Y^{s*} > 0 \wedge Y^{h*} \leq 0 \right] = \sigma_s^{ss} \lambda^{ss} + \sigma_s^{sh} \lambda^{sh}$$

$$(A40) \quad E \left[\tilde{\xi}_h^b | Y^{s*} > 0 \wedge Y^{h*} > 0 \right] = \sigma_h^{bs} \lambda^{bs} + \sigma_h^{bh} \lambda^{bh}$$

$$(A41) \quad E \left[\tilde{\xi}_h^h | Y^{s*} \leq 0 \wedge Y^{h*} > 0 \right] = \sigma_h^{hs} \lambda^{hs} + \sigma_h^{hh} \lambda^{hh}$$

where the λ s are defined as in equations (31) to (33) of the main article.

Proof of Selectivity Terms

In the following we derive the selectivity terms used in our 2SLS/IV estimation procedure.

To this end we consider a trivariate normal distribution of the variables X_1 , X_2 and X_3 with density function $\phi_3(X_1, X_2, X_3)$, mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$, where it holds:

$$(A42) \quad \boldsymbol{\mu} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}; \quad \boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ & 1 & \rho \\ & & 1 \end{pmatrix}$$

The corresponding marginal normal distributions of the variables X_2 and X_3 are bivariate normal distributed with density function $\phi_2(X_1, X_2)$, mean vector $\boldsymbol{\mu}_{23}$ and covariance matrix $\boldsymbol{\Sigma}_{23}$, where it holds (see for example [Greene 2003](#)):

$$(A43) \quad \boldsymbol{\mu}_{23} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}; \quad \boldsymbol{\Sigma}_{23} = \begin{pmatrix} 1 & \rho \\ & 1 \end{pmatrix}$$

The corresponding conditional distribution of X_1 has density function $\phi(X_1 | X_2, X_3)$, mean μ_1^* , and variance σ_1^{2*} , where it holds (see for example [Greene 2003](#)):

$$(A44) \quad \mu_1^* = \frac{(\sigma_{12} - \rho\sigma_{13})X_2 + (\sigma_{13} - \rho\sigma_{12})X_3}{1 - \rho^2}$$

$$(A45) \quad \sigma_1^{2*} = \sigma_1^2 - \frac{\sigma_{12}^2 - 2\sigma_{12}\sigma_{13}\rho + \sigma_{13}^2}{1 - \rho^2}$$

Given the definitions above we first prove the following three Lemmas

Lemma 1:

For $a_2, a_3 \in \mathbb{R}$ it holds

$$(A46) \quad \int_{a_3}^{\infty} \phi(X_3) \phi\left(\frac{a_2 - \rho X_3}{\sqrt{1 - \rho^2}}\right) dX_3 = \sqrt{1 - \rho^2} \phi(a_2) \Phi\left(\frac{-a_3 + \rho a_2}{\sqrt{1 - \rho^2}}\right)$$

Proof:

$$\int_{a_3}^{\infty} \phi(X_3) \phi\left(\frac{a_2 - \rho X_3}{\sqrt{1 - \rho^2}}\right) dX_3$$

$$(A47) \quad = \int_{a_3}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}X_3^2} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{a_2 - \rho X_3}{\sqrt{1-\rho^2}}\right)^2} dX_3$$

$$(A48) \quad = \int_{a_3}^{\infty} \frac{1}{2\pi} e^{-\frac{1}{2}\left(X_3^2 + \frac{(a_2 - \rho X_3)^2}{1-\rho^2}\right)} dX_3$$

$$(A49) \quad = \int_{a_3}^{\infty} \frac{1}{2\pi} e^{-\frac{1}{2}\left(\frac{X_3^2(1-\rho^2)}{1-\rho^2} + \frac{a_2^2 - 2a_2\rho X_3 + \rho^2 X_3^2}{1-\rho^2}\right)} dX_3$$

$$(A50) \quad = \int_{a_3}^{\infty} \frac{1}{2\pi} e^{-\frac{1}{2}\left(\frac{a_2^2(1-\rho^2)}{1-\rho^2} + \frac{X_3^2 - 2X_3\rho a_2 + \rho^2 a_2^2}{1-\rho^2}\right)} dX_3$$

$$(A51) \quad = \int_{a_3}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}a_2^2} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{X_3 - \rho a_2}{\sqrt{1-\rho^2}}\right)^2} dX_3$$

$$(A52) \quad = \int_{a_3}^{\infty} \phi(a_2) \phi\left(\frac{X_3 - \rho a_2}{\sqrt{1-\rho^2}}\right) dX_3$$

$$(A53) \quad = \phi(a_2) \sqrt{1-\rho^2} \int_{a_3}^{\infty} \frac{1}{\sqrt{1-\rho^2}} \phi\left(\frac{X_3 - \rho a_2}{\sqrt{1-\rho^2}}\right) dX_3$$

$$(A54) \quad = \phi(a_2) \sqrt{1-\rho^2} \int_{\frac{a_3 - \rho a_2}{\sqrt{1-\rho^2}}}^{\infty} \phi(Z_3) dZ_3$$

$$(A55) \quad = \sqrt{1-\rho^2} \phi(a_2) \Phi\left(\frac{-a_3 + \rho a_2}{\sqrt{1-\rho^2}}\right)$$

q.e.d.

Corollary to Lemma 1:

$$(A56) \quad \int_{-\infty}^{a_3} \phi(X_3) \phi\left(\frac{a_2 - \rho X_3}{\sqrt{1-\rho^2}}\right) dX_3 = \sqrt{1-\rho^2} \phi(a_2) \Phi\left(\frac{a_3 - \rho a_2}{\sqrt{1-\rho^2}}\right)$$

Lemma 2

For $a_2, a_3 \in \mathbb{R}$ it holds

$$(A57) \quad \begin{aligned} & \int_{a_3}^{\infty} X_3 \phi(X_3) \Phi\left(\frac{-a_2 + \rho X_3}{\sqrt{1-\rho^2}}\right) dX_3 \\ &= \phi(a_3) \Phi\left(\frac{-a_2 + \rho a_3}{\sqrt{1-\rho^2}}\right) + \rho \phi(a_2) \Phi\left(\frac{-a_3 + \rho a_2}{\sqrt{1-\rho^2}}\right) \end{aligned}$$

Proof:

$$\int_{a_3}^{\infty} X_3 \phi(X_3) \Phi\left(\frac{-a_2 + \rho X_3}{\sqrt{1-\rho^2}}\right) dX_3$$

$$(A58) \quad = \int_{a_3}^{\infty} g'(X_3) f(X_3) dX_3$$

with

$$(A59) \quad g'(X_3) = X_3 \phi(X_3)$$

$$(A60) \quad f(X_3) = \Phi\left(\frac{-a_2 + \rho X_3}{\sqrt{1 - \rho^2}}\right)$$

From partial integration it follows

$$(A61) \quad \int_{a_3}^{\infty} g'(X_3) f(X_3) dX_3 = \lim_{a \rightarrow \infty} g(a) f(a) - g(a_3) f(a_3) - \int_{a_3}^{\infty} g(X_3) f'(X_3) dX_3$$

with

$$(A62) \quad g(X_3) = -\phi(X_3)$$

$$(A63) \quad f'(X_3) = \phi\left(\frac{-a_2 + \rho X_3}{\sqrt{1 - \rho^2}}\right)$$

substituting (A62) and (A63) into (A61) we get

$$(A64) \quad \lim_{a \rightarrow \infty} g(a) f(a) - g(a_3) f(a_3) - \int_{a_3}^{\infty} g(X_3) f'(X_3) dX_3 = \phi(a_3) \Phi\left(\frac{-a_2 + \rho a_3}{\sqrt{1 - \rho^2}}\right) + \frac{\rho}{\sqrt{1 - \rho^2}} \int_{a_3}^{\infty} \phi(X_3) \phi\left(\frac{-a_2 + \rho X_3}{\sqrt{1 - \rho^2}}\right) dX_3$$

$$(A65) \quad = \phi(a_3) \Phi\left(\frac{-a_2 + \rho a_3}{\sqrt{1 - \rho^2}}\right) + \frac{\rho}{\sqrt{1 - \rho^2}} \int_{a_3}^{\infty} \phi(X_3) \phi\left(\frac{a_2 - \rho X_3}{\sqrt{1 - \rho^2}}\right) dX_3$$

applying Lemma 1 results in

$$(A66) \quad \phi(a_3) \Phi\left(\frac{-a_2 + \rho a_3}{\sqrt{1 - \rho^2}}\right) + \frac{\rho}{\sqrt{1 - \rho^2}} \int_{a_3}^{\infty} \phi(X_3) \phi\left(\frac{a_2 - \rho X_3}{\sqrt{1 - \rho^2}}\right) dX_3 = \phi(a_3) \Phi\left(\frac{-a_2 + \rho a_3}{\sqrt{1 - \rho^2}}\right) + \rho \phi(a_2) \Phi\left(\frac{-a_3 + \rho a_2}{\sqrt{1 - \rho^2}}\right)$$

q.e.d.

Corollary to Lemma 2:

$$(A67) \quad \int_{-\infty}^{a_3} X_3 \phi(X_3) \Phi\left(\frac{-a_2 + \rho X_3}{\sqrt{1 - \rho^2}}\right) dX_3 = -\phi(a_3) \Phi\left(\frac{-a_2 + \rho a_3}{\sqrt{1 - \rho^2}}\right) + \rho \phi(a_2) \Phi\left(\frac{a_3 - \rho a_2}{\sqrt{1 - \rho^2}}\right)$$

Lemma 3

For $a_2, a_3 \in \mathbb{R}$ it holds:

$$(A68) \quad \int_{-\infty}^{\infty} \int_{a_2}^{\infty} \int_{a_3}^{\infty} \phi_3(X_1, X_2, X_3) dX_3 dX_2 dX_1 = \Phi_2(-a_2, -a_3, \mathbf{\Sigma}_{23})$$

Corollary to Lemma 3:

$$(A69) \quad \int_{-\infty}^{\infty} \int_{a_2}^{\infty} \int_{-\infty}^{a_3} \phi_3(X_1, X_2, X_3) dX_3 dX_2 dX_1 = \Phi_2(-a_2, a_3, (1 - \rho^2) \mathbf{\Sigma}_{23}^{-1})$$

Lemma 4:

$$(A70) \quad \int X_2 \phi(X_2) dX_2 = -\phi(X_2)$$

Proof:

$$(A71) \quad \frac{\partial \phi(X_2)}{\partial X_2} = -X_2 \phi(X_2)$$

q.e.d.

Theorem

Given a trivariate normal distribution as defined above. Then it holds for any $a_2, a_3 \in \mathbb{R}$:

(i) $E(X_1 | X_2 > a_2 \wedge X_3 > a_3)$

$$(A72) \quad = \frac{\sigma_{13} \phi(a_3) \Phi\left(\frac{-a_2 + \rho a_3}{\sqrt{1 - \rho^2}}\right) + \sigma_{12} \phi(a_2) \Phi\left(\frac{-a_3 + \rho a_2}{\sqrt{1 - \rho^2}}\right)}{\Phi_2(-a_2, -a_3, \mathbf{\Sigma}_{23})}$$

(ii) $E(X_1 | X_2 > a_2 \wedge X_3 < a_3)$

$$(A73) \quad = \frac{-\sigma_{13} \phi(a_3) \Phi\left(\frac{-a_2 + \rho a_3}{\sqrt{1 - \rho^2}}\right) + \sigma_{12} \phi(a_2) \Phi\left(\frac{a_3 - \rho a_2}{\sqrt{1 - \rho^2}}\right)}{\Phi_2(-a_2, a_3, (1 - \rho^2) \mathbf{\Sigma}_{23}^{-1})}$$

Proof of (i):

It holds the definition

$$(A74) \quad E(X_1 | X_2 > a_2 \wedge X_3 > a_3) = \frac{\int_{-\infty}^{\infty} \int_{a_2}^{\infty} \int_{a_3}^{\infty} X_1 \phi_3(X_1, X_2, X_3) dX_3 dX_2 dX_1}{\int_{-\infty}^{\infty} \int_{a_2}^{\infty} \int_{a_3}^{\infty} \phi_3(X_1, X_2, X_3) dX_3 dX_2 dX_1}$$

Applying Lemma 3 results in

$$(A75) \quad E(X_1 | X_2 > a_2 \wedge X_3 > a_3) = \frac{\int_{-\infty}^{\infty} \int_{a_2}^{\infty} \int_{a_3}^{\infty} X_1 \phi_3(X_1, X_2, X_3) dX_3 dX_2 dX_1}{\Phi_2(-a_2, -a_3, \mathbf{\Sigma}_{23})}$$

Now it holds for any trivariate normal distribution

$$(A76) \quad \int_{-\infty}^{\infty} \int_{a_2}^{\infty} \int_{a_3}^{\infty} X_1 \phi_3(X_1, X_2, X_3) dX_3 dX_2 dX_1 \\ = \int_{a_2}^{\infty} \int_{a_3}^{\infty} \phi_2(X_2, X_3) \int_{-\infty}^{\infty} X_1 \phi_3(X_1 | X_2, X_3) dX_1 dX_3 dX_2$$

$$(A77) \quad = \int_{a_2}^{\infty} \int_{a_3}^{\infty} \phi_2(X_2, X_3) \mu_1^* dX_3 dX_2$$

$$(A78) \quad = \int_{a_2}^{\infty} \int_{a_3}^{\infty} \phi_2(X_2, X_3) \frac{(\sigma_{12} - \rho\sigma_{13})X_2 + (\sigma_{13} - \sigma_{12}\rho)X_3}{1 - \rho^2} dX_3 dX_2$$

$$(A79) \quad = \int_{a_2}^{\infty} \int_{a_3}^{\infty} \phi(X_3) \phi(X_2 | X_3) (K_2 X_2 + K_3 X_3) dX_3 dX_2$$

with

$$(A80) \quad K_2 = \frac{\sigma_{12} - \rho\sigma_{13}}{1 - \rho^2}$$

$$(A81) \quad K_3 = \frac{\sigma_{13} - \rho\sigma_{12}}{1 - \rho^2}$$

Now it holds

$$(A82) \quad \int_{a_2}^{\infty} \int_{a_3}^{\infty} \phi(X_3) \frac{1}{\sqrt{1 - \rho^2}} \phi\left(\frac{X_2 - \rho X_3}{\sqrt{1 - \rho^2}}\right) (K_2 X_2 + K_3 X_3) dX_3 dX_2 \\ = \int_{a_3}^{\infty} \phi(X_3) \left(K_2 \int_{a_2}^{\infty} X_2 \frac{1}{\sqrt{1 - \rho^2}} \phi\left(\frac{X_2 - \rho X_3}{\sqrt{1 - \rho^2}}\right) dX_2 \right. \\ \left. + K_3 X_3 \int_{a_2}^{\infty} \frac{1}{\sqrt{1 - \rho^2}} \phi\left(\frac{X_2 - \rho X_3}{\sqrt{1 - \rho^2}}\right) dX_2 \right) dX_3$$

$$(A83) \quad = \int_{a_3}^{\infty} \phi(X_3) \left(K_2 \sqrt{1 - \rho^2} \int_{a_2}^{\infty} \frac{1}{\sqrt{1 - \rho^2}} \frac{X_2 - \rho X_3}{\sqrt{1 - \rho^2}} \phi\left(\frac{X_2 - \rho X_3}{\sqrt{1 - \rho^2}}\right) dX_2 \right. \\ \left. + (K_2 \rho + K_3) X_3 \int_{a_2}^{\infty} \frac{1}{\sqrt{1 - \rho^2}} \phi\left(\frac{X_2 - \rho X_3}{\sqrt{1 - \rho^2}}\right) dX_2 \right) dX_3$$

$$(A84) \quad = \int_{a_3}^{\infty} \phi(X_3) \left(K_2 \sqrt{1 - \rho^2} \int_{\frac{a_2 - \rho X_3}{\sqrt{1 - \rho^2}}}^{\infty} Z_2 \phi(Z_2) dZ_2 \right. \\ \left. + (K_2 \rho + K_3) X_3 \int_{\frac{a_2 - \rho X_3}{\sqrt{1 - \rho^2}}}^{\infty} \phi(Z_2) dZ_2 \right) dX_3$$

applying Lemma 4 we get

$$\int_{a_3}^{\infty} \phi(X_3) K_2 \sqrt{1 - \rho^2} \phi\left(\frac{a_2 - \rho X_3}{\sqrt{1 - \rho^2}}\right) dX_3$$

$$\begin{aligned}
& + \int_{a_3}^{\infty} \phi(X_3) (K_2\rho + K_3) X_3 \Phi\left(\frac{-a_2 + \rho X_3}{\sqrt{1-\rho^2}}\right) \\
\text{(A85)} \quad & = K_2 \sqrt{1-\rho^2} \int_{a_3}^{\infty} \phi(X_3) \phi\left(\frac{a_2 - \rho X_3}{\sqrt{1-\rho^2}}\right) dX_3 \\
& + (K_2\rho + K_3) \int_{a_3}^{\infty} X_3 \phi(X_3) \Phi\left(\frac{-a_2 + \rho X_3}{\sqrt{1-\rho^2}}\right) dX_3
\end{aligned}$$

applying Lemma 1 and Lemma 2 we get

$$\begin{aligned}
& K_2 \sqrt{1-\rho^2} \sqrt{1-\rho^2} \phi(a_2) \Phi\left(\frac{-a_3 + \rho a_2}{\sqrt{1-\rho^2}}\right) \\
& + (K_2\rho + K_3) \left(\phi(a_3) \Phi\left(\frac{-a_2 + \rho a_3}{\sqrt{1-\rho^2}}\right) + \rho \phi(a_2) \Phi\left(\frac{-a_3 + \rho a_2}{\sqrt{1-\rho^2}}\right) \right) \\
\text{(A86)} \quad & = (K_2(1-\rho^2) + (K_2\rho + K_3)\rho) \phi(a_2) \Phi\left(\frac{-a_3 + \rho a_2}{\sqrt{1-\rho^2}}\right)
\end{aligned}$$

$$\begin{aligned}
& + (K_2\rho + K_3) \phi(a_3) \Phi\left(\frac{-a_2 + \rho a_3}{\sqrt{1-\rho^2}}\right) \\
\text{(A87)} \quad & = (K_2 + K_3\rho) \phi(a_2) \Phi\left(\frac{-a_3 + \rho a_2}{\sqrt{1-\rho^2}}\right) \\
& + (K_2\rho + K_3) \phi(a_3) \Phi\left(\frac{-a_2 + \rho a_3}{\sqrt{1-\rho^2}}\right)
\end{aligned}$$

substituting (A81) and (A80) for K_2 and K_3

$$\begin{aligned}
& \left(\frac{\sigma_{12} - \rho\sigma_{13}}{1-\rho^2} + \frac{\sigma_{13} - \rho\sigma_{12}}{1-\rho^2} \rho \right) \phi(a_2) \Phi\left(\frac{-a_3 + \rho a_2}{\sqrt{1-\rho^2}}\right) \\
& + \left(\frac{\sigma_{12} - \rho\sigma_{13}}{1-\rho^2} \rho + \frac{\sigma_{13} - \rho\sigma_{12}}{1-\rho^2} \right) \phi(a_3) \Phi\left(\frac{-a_2 + \rho a_3}{\sqrt{1-\rho^2}}\right) \\
\text{(A88)} \quad & = \left(\frac{\sigma_{12} - \rho\sigma_{13} + \sigma_{13}\rho - \rho^2\sigma_{12}}{1-\rho^2} \right) \phi(a_2) \Phi\left(\frac{-a_3 + \rho a_2}{\sqrt{1-\rho^2}}\right) \\
& + \left(\frac{\rho\sigma_{12} - \rho^2\sigma_{13} + \sigma_{13} - \rho\sigma_{12}}{1-\rho^2} \right) \phi(a_3) \Phi\left(\frac{-a_2 + \rho a_3}{\sqrt{1-\rho^2}}\right)
\end{aligned}$$

$$\text{(A89)} \quad = \sigma_{12} \phi(a_2) \Phi\left(\frac{-a_3 + \rho a_2}{\sqrt{1-\rho^2}}\right) + \sigma_{13} \phi(a_3) \Phi\left(\frac{-a_2 + \rho a_3}{\sqrt{1-\rho^2}}\right)$$

q.e.d.

Proof of (ii):

This proof is analogous to the proof of (i) except that the Corollaries are applied in place of the Lemmas.

Formulas to Calculate Farm-Household Elasticities

Notations

Price Elasticities on Production Side

$$\epsilon_{ij} = \frac{\partial X_i P_j}{\partial P_j X_i} = \text{traditional price elasticity of netput } i \text{ with respect to price of netput } j$$

$$\epsilon_{ij}^{FHM} = \frac{\partial X_i P_j}{\partial P_j X_i} = \text{FHM price elasticity of netput } i \text{ with respect to price of netput/good } j$$

Price Elasticities on Consumption Side

$$\Theta_{ij} = \frac{\partial C_i P_j}{\partial P_j C_i} = \text{traditional Marshallian price elasticity of good } i \text{ with respect to price of good } j$$

$$\Theta_{ij}^H = \frac{\partial C_i^H P_j}{\partial P_j C_i} = \text{traditional Hicksian price elasticity of good } i \text{ with respect to price of good } j$$

$$\eta_i = \frac{\partial C_i Y}{\partial Y C_i} = \text{traditional income elasticity of good } i$$

$$\Theta_{ij}^{FHM} = \frac{\partial C_i P_j}{\partial P_j C_i} = \text{FHM price elasticity of good } i \text{ with respect to price of netput/good } j$$

Price Elasticities of Labor Allocation

$$\phi_{sL} = \frac{\partial X_L^s P_L^s}{\partial P_L^s X_L^s} = \text{traditional price elasticity of supplied labor with respect to labor price}$$

$$\phi_{hL} = \frac{\partial X_L^h P_L^h}{\partial P_L^h X_L^h} = \text{traditional price elasticity of hired labor with respect to labor price}$$

$$\phi_{sj}^{FHM} = \frac{\partial X_L^s P_j}{\partial P_j X_L^s} = \text{FHM price elasticity of supplied labor with respect to price of netput/good } j$$

$$\phi_{hj}^{FHM} = \frac{\partial X_L^h P_j}{\partial P_j X_L^h} = \text{FHM price elasticity of hired labor with respect to price of netput/good } j$$

$$\phi_{nj}^{FHM} = \frac{\partial X_L^n P_j}{\partial P_j X_L^n} = \text{FHM price elasticity of net supplied labor with respect to price of netput/good } j$$

$\varphi_{fj}^{FHM} = \frac{\partial X_L^f}{\partial P_j} \frac{P_j}{X_L^f} =$ FHM price elasticity of family labor on the farm with respect to price of netput/good j

Shadow Price Elasticity of Labor

$\Psi_j = \frac{\partial P_L^*}{\partial P_j} \frac{P_j}{P_L^*} =$ elasticity of the shadow price of labor with respect to price of netput/good j

Price Elasticities of the Separable Household Models

Price Elasticities on Production Side

The price elasticities on production side are simply the traditional price elasticities:

$$(A90) \quad \varepsilon_{ij}^{FHM} = \varepsilon_{ij} \quad \forall i, j \in \{a, c, v, L\}$$

$$(A91) \quad \varepsilon_{im}^{FHM} = 0 \quad \forall i \in \{a, c, v, L\}$$

Price Elasticities on Consumption Side

The price elasticities on consumption side consist of the normal Marshallian price effect and of an income effect due to an income change from farming or from working off-farm:

$$(A92) \quad \Theta_{ij}^{sFHM} = \left. \frac{\partial C_i}{\partial P_j} \right|_{Y=\text{const.}} \frac{P_j}{C_i} + \frac{\partial C_i}{\partial Y} \frac{\partial Y}{\partial P_j} \frac{P_j}{C_i}$$

$$(A93) \quad = \frac{\partial C_i^H}{\partial P_j} \frac{P_j}{C_i} + \frac{\partial C_i}{\partial Y} \frac{Y}{C_i} \left(\frac{\partial Y}{\partial P_j} - C_j \right) \frac{P_j}{Y}$$

$$(A94) \quad = \Theta_{ij}^H + \eta_i \left(\frac{\partial Y}{\partial P_j} - C_j \right) \frac{P_j}{Y}$$

Evaluating $\frac{\partial Y}{\partial P_j}$ and removing all terms that are zero, we get the elasticities for each of the prices:

$$(A95) \quad \Theta_{ij}^{sFHM} = \eta_i \frac{P_j X_j}{Y} \quad \forall i \in \{m, a, L\}, j \in \{c, v\}$$

$$(A96) \quad \Theta_{ia}^{sFHM} = \Theta_{ia}^H + \eta_i \frac{P_a (X_a - C_a)}{Y} \quad \forall i \in \{m, a, L\}$$

$$(A97) \quad \Theta_{iL}^{sFHM} = \Theta_{iL}^H + \eta_i \frac{P_j (X_L^s - X_L^h)}{Y} \quad \forall i \in \{m, a, L\}$$

$$(A98) \quad \Theta_{im}^{sFHM} = \Theta_{im}^H - \eta_i \frac{P_m C_m}{Y} \quad \forall i \in \{m, a, L\}$$

Price Elasticity of Net Supply of Labor

The price elasticity of net supply of labor is calculated residually:

$$(A99) \quad \varphi_{nj}^{sFHM} = \frac{\partial (X_L^s - X_L^h)}{\partial P_j} \frac{P_j}{X_L^{sn}}$$

$$(A100) \quad = \frac{\partial (T_L + X_L - C_L)}{\partial P_j} \frac{P_j}{X_L^n}$$

$$(A101) \quad = \frac{\partial X_L}{\partial P_j} \frac{P_j}{X_L} \frac{X_L}{X_L^n} - \frac{\partial C_L}{\partial P_j} \frac{P_j}{C_L} \frac{C_L}{X_L^n}$$

$$(A102) \quad = \varepsilon_{Lj}^{FHM} \frac{X_L}{X_L^n} - \Theta_{Lj}^{FHM} \frac{C_L}{X_L^n} \quad \forall j \in \{a, c, v, L, m\}$$

Price Elasticities of the Non-separable Household Models

The following formulas are valid for all four labor regimes. In case that the household does not supply labor, X_L^s and φ_L^s have to be set to zero and in case that the household does not hire labor, X_L^h and φ_L^h have to be set to zero.

Shadow Price Elasticities

We derive the shadow price elasticities from equation (14) of the main article:

$$(A103) \quad \Psi_j = \frac{-\frac{\partial X_L}{\partial P_j} + \frac{\partial C_L}{\partial P_j} \Big|_{Y=\text{const.}} + \frac{\partial C_L}{\partial Y} \frac{\partial Y}{\partial P_j} \frac{P_j}{P_L^*}}{\frac{\partial X_L}{\partial P_L^*} + \frac{\partial X_L^h}{\partial P_L^*} - \frac{\partial X_L^s}{\partial P_L^*} - \frac{\partial C_L^H}{\partial P_L^*}} \frac{P_j}{P_L^*}$$

$$(A104) \quad = \frac{-\frac{\partial X_L}{\partial P_j} + \frac{\partial C_L^H}{\partial P_j} + \frac{\partial C_L}{\partial Y} \left(\frac{\partial Y}{\partial P_j} - C_j \right)}{\frac{\partial X_L}{\partial P_L^*} + \frac{\partial X_L^h}{\partial P_L^*} - \frac{\partial X_L^s}{\partial P_L^*} - \frac{\partial C_L^H}{\partial P_L^*}} \frac{P_j}{P_L^*}$$

$$(A105) \quad = \frac{-\frac{\partial X_L}{\partial P_j} \frac{P_j}{X_L} X_L + \frac{\partial C_L^H}{\partial P_j} \frac{P_j}{C_L} C_L + \frac{\partial C_L}{\partial Y} \frac{Y}{C_L} \left(\frac{\partial Y}{\partial P_j} - C_j \right) \frac{P_j}{Y} C_L}{\frac{\partial X_L}{\partial P_L^*} \frac{P_L^*}{X_L} X_L + \frac{\partial X_L^h}{\partial P_L^*} \frac{P_L^*}{X_L^h} X_L^h - \frac{\partial X_L^s}{\partial P_L^*} \frac{P_L^*}{X_L^s} X_L^s - \frac{\partial C_L^H}{\partial P_L^*} \frac{P_L^*}{C_L} C_L}$$

$$(A106) \quad = \frac{-\varepsilon_{Lj} X_L + \Theta_{Lj}^H C_L + \eta_L \left(\frac{\partial Y}{\partial P_j} - C_j \right) \frac{P_j}{Y} C_L}{\varepsilon_{LL} X_L + \varphi_L^h X_L^h - \varphi_L^s X_L^s - \Theta_{LL}^H C_L}$$

Evaluating $\frac{\partial Y}{\partial P_j}$ and removing all terms that are zero, we get the elasticities for each of the exogenous prices:

$$(A107) \quad \Psi_j = \frac{-\varepsilon_{Lj}X_L + \eta_L \frac{P_j X_j}{Y} C_L}{\varepsilon_{LL}X_L + \varphi_L^h X_L^h - \varphi_L^s X_L^s - \Theta_{LL}^H C_L} \quad \forall j \in \{c, v\}$$

$$(A108) \quad \Psi_a = \frac{-\varepsilon_{La}X_L + \Theta_{La}^H C_L + \eta_L \frac{P_a (X_a - C_a)}{Y} C_L}{\varepsilon_{LL}X_L + \varphi_L^h X_L^h - \varphi_L^s X_L^s - \Theta_{LL}^H C_L}$$

$$(A109) \quad \Psi_m = \frac{\Theta_{Lm}^H C_L - \eta_L \frac{P_L C_L}{Y} C_L}{\varepsilon_{LL}X_L + \varphi_L^h X_L^h - \varphi_L^s X_L^s - \Theta_{LL}^H C_L}$$

Given the convexity of the profit function $\Pi(\cdot)$ in netput prices and the concavity of the expenditure function $e(\cdot)$ in commodity prices and assuming that $g(\cdot)$ is convex in X_L^h and $f(\cdot)$ is concave in X_L^s , the denominator is always positive, because $\varphi_L^h = \left(\partial^2 g / \partial X_L^{h2}\right)^{-1} (P_L^h / X_L^h) \geq 0$, $X_L^h \geq 0$, $\varepsilon_{LL} = \left(\partial^2 \Pi / \partial P_L^2\right) (P_L / X_L) \leq 0$, $X_L \leq 0$, $\varphi_L^s = \left(\partial^2 f / \partial X_L^{s2}\right)^{-1} (P_L^s / X_L^s) \leq 0$, $X_L^s \geq 0$, $\Theta_{LL}^H = \left(\partial^2 e / \partial P_L^2\right) (C_L / P_L) \leq 0$, and $C_L \geq 0$.

Price Elasticities on Production Side

We derive the price elasticities on production side from equation (13) of the main article:

$$(A110) \quad \varepsilon_{ij}^{iFHM} = \left. \frac{\partial X_i}{\partial P_j} \right|_{P_L^* = \text{const.}} \frac{P_j}{X_i} + \frac{\partial X_i}{\partial P_L^*} \frac{P_L^*}{X_i} \frac{\partial P_L^*}{\partial P_j} \frac{P_j}{P_L}$$

$$(A111) \quad = \varepsilon_{ij}^{sFHM} + \varepsilon_{iL} \Psi_j$$

Substituting the direct component, which is the price elasticity of the separable model ε_{ij}^{sFHM} , we get the elasticities for each of the exogenous prices:

$$(A112) \quad \varepsilon_{ij}^{iFHM} = \varepsilon_{ij} + \varepsilon_{iL} \Psi_j \quad \forall i \in \{a, c, v, L\}, j \in \{c, a, v\}$$

$$(A113) \quad \varepsilon_{im}^{iFHM} = \varepsilon_{iL} \Psi_m \quad \forall i \in \{a, c, v, L\}$$

Price Elasticities on Consumption Side

We derive the price elasticities on consumption side from equation (13) of the main article:

$$(A114) \quad \Theta_{ij}^{iFHM} = \left. \frac{\partial C_i}{\partial P_j} \right|_{P_L^* = \text{const.}} \frac{P_j}{C_i} + \frac{\partial C_i}{\partial P_L^*} \frac{\partial P_L^*}{\partial P_j} \frac{P_j}{C_i}$$

$$(A115) \quad = \left. \frac{\partial C_i}{\partial P_j} \right|_{P_L^* = \text{const.}} \frac{P_j}{C_i} + \left(\left. \frac{\partial C_i}{\partial P_L^*} \right|_{Y = \text{const.}} + \frac{\partial C_i}{\partial Y} \frac{\partial Y}{\partial P_L^*} \right) \frac{\partial P_L^*}{\partial P_j} \frac{P_j}{C_i}$$

$$(A116) \quad = \frac{\partial C_i}{\partial P_j} \Big|_{P_L^* = \text{const.}} \frac{P_j}{C_i} + \frac{\partial C_i^H}{\partial P_L^*} \frac{P_L^*}{C_i} \frac{\partial P_L^*}{\partial P_j} \frac{P_j}{P_L^*}$$

$$(A117) \quad = \Theta_{ij}^{sFHM} + \Theta_{iL}^H \Psi_j$$

Substituting the direct component, which is the price elasticity of the separable model Θ_{ij}^{sFHM} , we get the elasticities for each of the exogenous prices:

$$(A118) \quad \Theta_{ij}^{iFHM} = \eta_i \frac{P_j X_j}{Y} + \Theta_{iL}^H \Psi_j \quad \forall i \in \{m, a, L\}, j \in \{c, v\}$$

$$(A119) \quad \Theta_{ia}^{iFHM} = \Theta_{ia}^H + \eta_i \frac{P_a (X_a - C_a)}{Y} + \Theta_{iL}^H \Psi_a \quad \forall i \in \{m, a, L\}$$

$$(A120) \quad \Theta_{im}^{iFHM} = \Theta_{im}^H - \eta_i \frac{P_m C_m}{Y} + \Theta_{iL}^H \Psi_m \quad \forall i \in \{m, a, L\}$$

Price Elasticities of Labor Allocation

We derive the price elasticities of labor supply and demand from equation (13) of the main article. Since the labor supply and demand do not directly depend on the exogenous prices, the direct component is zero:

$$(A121) \quad \varphi_{sj}^{iFHM} = \frac{\partial X_L^s P_L^*}{\partial P_L^* X_L^s} \frac{\partial P_L^*}{\partial P_j} \frac{P_j}{P_L^*}$$

$$(A122) \quad = \varphi_L^s \Psi_j \quad \forall j \in \{c, a, v, m\}$$

$$(A123) \quad \varphi_{hj}^{iFHM} = \frac{\partial X_L^h P_L^*}{\partial P_L^* X_L^h} \frac{\partial P_L^*}{\partial P_j} \frac{P_j}{P_L^*}$$

$$(A124) \quad = \varphi_L^h \Psi_j \quad \forall j \in \{c, a, v, m\}$$

The remaining labor allocation elasticities are calculated residually:

$$(A125) \quad \varphi_{nj}^{iFHM} = \frac{\partial (X_L^s - X_L^h)}{\partial P_L^*} \frac{P_L^*}{X_L^n} \frac{\partial P_L^*}{\partial P_j} \frac{P_j}{P_L^*}$$

$$(A126) \quad = \frac{\partial X_L^s P_L^*}{\partial P_L^* X_L^s} \frac{\partial P_L^*}{\partial P_j} \frac{P_j}{P_L^*} \frac{X_L^s}{X_L^n} - \frac{\partial X_L^h P_L^*}{\partial P_L^* X_L^h} \frac{\partial P_L^*}{\partial P_j} \frac{P_j}{P_L^*} \frac{X_L^s}{X_L^n}$$

$$(A127) \quad = \varphi_j^s \frac{X_L^s}{X_L^n} - \varphi_j^h \frac{X_L^h}{X_L^n} \quad j \in \{c, a, v, m\}$$

$$(A128) \quad \varphi_{fj}^{iFHM} = \frac{\partial (T_L - X_L^s - C_L^h)}{\partial P_L^*} \frac{P_L^*}{X_L^f} \frac{\partial P_L^*}{\partial P_j} \frac{P_j}{P_L^*}$$

$$(A129) \quad = -\frac{\partial X_L^s P_L^*}{\partial P_L^* X_L^s} \frac{\partial P_L^*}{\partial P_j} \frac{P_j}{P_L^*} \frac{X_L^s}{X_L^f} - \frac{\partial C_L P_L^*}{\partial P_L^* C_L} \frac{\partial P_L^*}{\partial P_j} \frac{P_j}{P_L^*} \frac{C_L}{X_L^f}$$

$$(A130) \quad = -\varphi_j^s \frac{X_L^s}{X_L^f} - \Theta_{Lj}^{iFHM} \frac{C_L}{X_L^f} \quad j \in \{c, a, v, m\}$$

Data Description

Table A2. Characteristics of the Sample

Variable	Unit	Mean	Minimum	Maximum	Std.deviation
N_k	number	1.3	0.0	5.0	1.2
N_w	number	2.8	0.0	7.0	1.3
N_o	number	0.7	0.0	3.0	0.8
A_h	years	43	20	76	11
T_L	hours	11399	3650	27375	4457
$ X_L $	hours	3686	400	9843	1717
X_L^h	hours	211	0	2085	365
X_L^s	hours	446	0	4000	876
X_L^n	hours	235	-2085	4000	1002
X_L^f	hours	3475	400	9236	1705
C_L	hours	7478	23	20873	4007
$P_m C_m$	1000 PLZ	91469	26365	280176	42853
$P_a C_a$	1000 PLZ	19041	1625	41853	7606
$P_c X_c$	1000 PLZ	132258	10451	1189412	133724
$P_a X_a$	1000 PLZ	212570	2669	2526524	239835
$P_v X_v $	1000 PLZ	211960	13480	2204671	213479
R_g	ha	14.7	1.2	101.5	12.4
R_k	1000 PLZ	649191	43960	4492025	554120
R_k/R_g	1000 PLZ / ha	46921	9170	215652	29039
N_c	number	0.9	0.0	3.0	0.6
W_u	%	19	9	25	4
W_i	km/100 km ²	58	39	71	9
W_t	1/1000 population	48	31	60	9
W_r	%	45	29	58	10
\tilde{P}_L	Poland = 100	88	73	115	13
P_L^*	1000 PLZ/h	38	6	230	28

Note: Calculations are based on [IERiGZ \(1995\)](#). PLZ = Polish Zloty. Variables: N_k = number of family members up to 14 years, N_w = number of family members between 15 and 60 years, N_o = number of family members older than 60 years, A_h = age of the household head, T_L = total time available, $|X_L|$ = labor input on the farm, X_L^h = hired labor, X_L^s = supplied labor, X_L^n = net supplied labor, X_L^f = family labor input on the farm, C_L = leisure, $P_m C_m$ = value of consumed market goods, $P_a C_a$ = value of consumed self-produced goods, $P_c X_c$ = value of produced crop products, $P_a X_a$ = value of produced animal products, $P_v |X_v|$ = value of utilized variable inputs, R_g = amount of land of the farm, R_k = amount of capital of the farm, N_c = number of cars owned by the household, W_u = regional unemployment rate, W_i = regional density of the road and railroad network, W_t = regional density of telephones, W_r = proportion of the population that lives in rural areas, \tilde{P}_L = relative average regional wage level, P_L^* = endogenous shadow price of labor.

Table A3. Characteristics of the Different Labor Regimes

Variable	Unit	All	Sup. & Dem.	Only Sup.	Only Dem.	Autarkic
Number		199	57	47	61	34
N_k	number	1.3	1.5	1.3	1.4	0.7
N_w	number	2.8	2.8	3.2	2.4	3.0
N_o	number	0.7	0.6	0.6	0.8	0.7
A_h	years	43	41	44	43	45
T_L	hours	11399	11110	12891	10082	12185
$ X_L $	hours	3686	3579	3372	4040	3668
X_L^h	hours	211	278	0	430	0
X_L^s	hours	446	515	1266	0	0
X_L^n	hours	235	237	1266	-430	0
X_L^f	hours	3475	3301	3372	3610	3668
C_L	hours	7478	7295	8254	6473	8517
$P_m C_m$	1000 PLZ	91469	105939	78012	97792	74467
$P_a C_a$	1000 PLZ	19041	18487	19245	19939	18076
$P_c X_c$	1000 PLZ	132258	157581	65883	180020	95869
$P_a X_a$	1000 PLZ	212570	220643	123997	300046	164531
$P_v X_v $	1000 PLZ	211960	232143	117552	299629	151343
R_g	ha	14.7	16.9	9.4	18.3	11.7
R_k	1000 PLZ	649191	788881	425398	816534	424132
R_k/R_g	1000 PLZ / ha	46921	49666	48516	48134	37938
N_c	number	0.9	1.0	0.8	0.9	0.8
W_u	%	19	20	19	18	20
W_i	km/100 km ²	58	55	60	60	57
W_t	1/1000 popul.	48	47	49	49	47
W_r	%	45	44	50	43	46
\tilde{P}_L	Poland = 100	88	85	90	89	88
P_L^*	1000 PLZ/h	38	46	30	44	28

Note: Calculations are based on [IERiGZ \(1995\)](#). PLZ = Polish Zloty. Variables: N_k = number of family members up to 14 years, N_w = number of family members between 15 and 60 years, N_o = number of family members older than 60 years, A_h = age of the household head, T_L = total time available, $|X_L|$ = labor input on the farm, X_L^h = hired labor, X_L^s = supplied labor, X_L^n = net supplied labor, X_L^f = family labor input on the farm, C_L = leisure, $P_m C_m$ = value of consumed market goods, $P_a C_a$ = value of consumed self-produced goods, $P_c X_c$ = value of produced crop products, $P_a X_a$ = value of produced animal products, $P_v |X_v|$ = value of utilized variable inputs, R_g = amount of land of the farm, R_k = amount of capital of the farm, N_c = number of cars owned by the household, W_u = regional unemployment rate, W_i = regional density of the road and railroad network, W_t = regional density of telephones, W_r = proportion of the population that lives in rural areas, \tilde{P}_L = relative average regional wage level, P_L^* = endogenous shadow price of labor.

Estimation Results

First-Stage Profit Function

Table A4. Estimation Results of the Unrestricted 1st-Stage Profit Function

Parameter	$i = c$		$i = a$		$i = v$	
	Coef.	(t-val)	Coef.	(t-val)	Coef.	(t-val)
α_i	-1.72	(-0.73)	20.1	(4.31)	-17.4	(-5.14)
β_{ic}	-14.8	(-1.12)	19.8	(2.68)	-4.92	(-0.37)
β_{ia}	19.8	(2.68)	61.6	(5.76)	-81.4	(-8.04)
β_{iv}	-4.92	(-0.37)	-81.4	(-8.04)	86.3	(5.08)
δ_{ig}	6258	(11.37)	1002	(0.93)	-4306	(-5.37)
δ_{ik}	0.0829	(5.77)	0.209	(7.47)	-0.111	(-5.36)
γ_{gg}	-1157392	(-6.45)				
γ_{gk}	36.7	(7.59)				
γ_{kk}	$-1.26 \cdot 10^{-3}$	(-9.79)				
R^2	0.709		0.286		0.685	

Note: For definitions of the estimated coefficients see equation (20) of the main article, where the subscripts c , a , v , g , and k indicate crop products, animal products, variable inputs, land, and capital, respectively. The standard errors of the coefficients that have not been directly estimated are calculated with the formula of Klein (1953, p. 258). Monotonicity is fulfilled at 100% of the observations.

Table A5. Estimation Results of the 1st-Stage Profit Function with Convexity Imposed

Parameter	$i = c$		$i = a$		$i = v$	
	Coef.	(t-val)	Coef.	(t-val)	Coef.	(t-val)
α_i	-2.28	(-0.57)	20.3	(3.16)	-17.0	(-3.21)
β_{ic}	3.31	(0.81)	14.6	(2.34)	-17.9	(-1.99)
β_{ia}	14.6	(2.34)	64.7	(2.93)	-79.3	(-3.16)
β_{iv}	-17.9	(-1.99)	-79.3	(-3.16)	97.3	(3.30)
δ_{ig}	6170	(4.60)	1024	(0.59)	-4294	(-2.26)
δ_{ik}	0.0855	(2.92)	0.208	(4.81)	-0.110	(-3.87)
γ_{gg}	-1149343	(-1.72)				
γ_{gk}	36.6	(1.89)				
γ_{kk}	$-1.26 \cdot 10^{-3}$	(-2.26)				
R^2	0.708		0.283		0.686	

Note: For definitions of the estimated coefficients see equation (20) of the main article, where the subscripts c , a , v , g , and k indicate crop products, animal products, variable inputs, land, and capital, respectively. The standard errors of the coefficients are calculated using the bootstrap resampling method (Efron 1979; Efron and Tibshirani 1993). Monotonicity is fulfilled at 100% of the observations. The R^2 values are almost identical to the model without convexity imposed, indicating that the data do not unreasonably contradict the convexity constraint (see table A4).

Shadow Prices of Labor

One estimated shadow price is negative. The other shadow prices have a mean of 38498 PLZ/h and a median of 30236 PLZ/h. In 1994 the average gross wage in Poland was 32820 PLZ/h. 68% of the estimated shadow prices deviate less than 50% from this value.

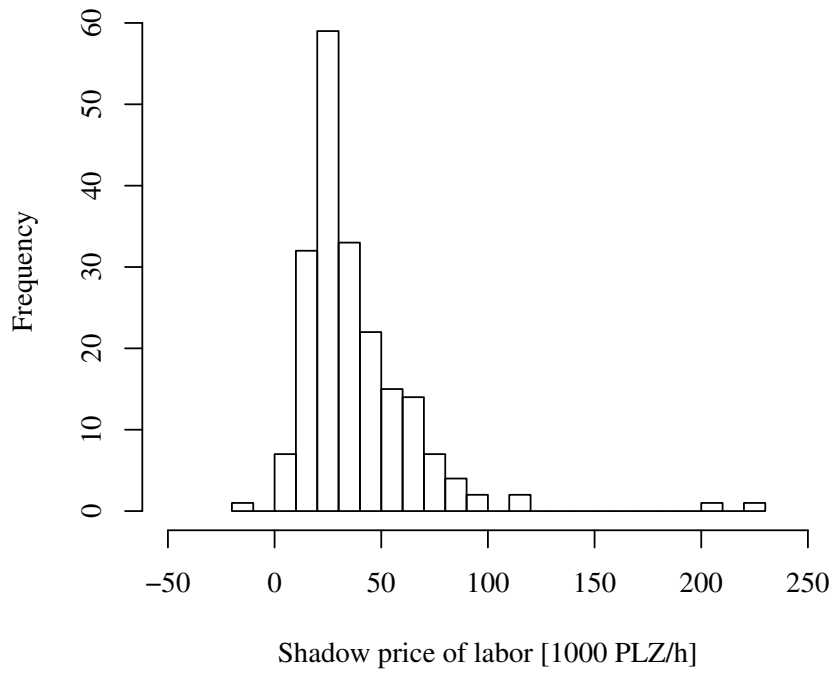


Figure A1. Distribution of the estimated shadow prices of labor

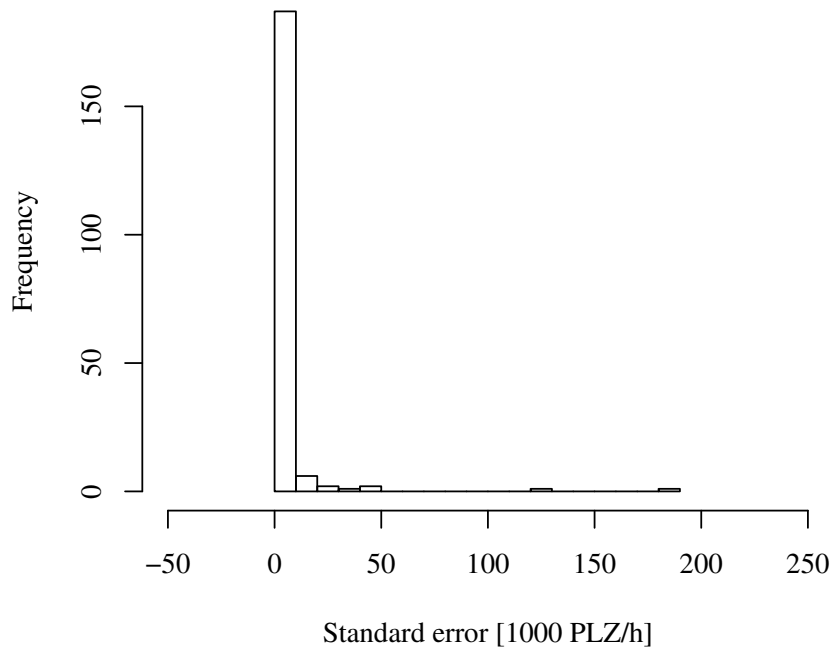
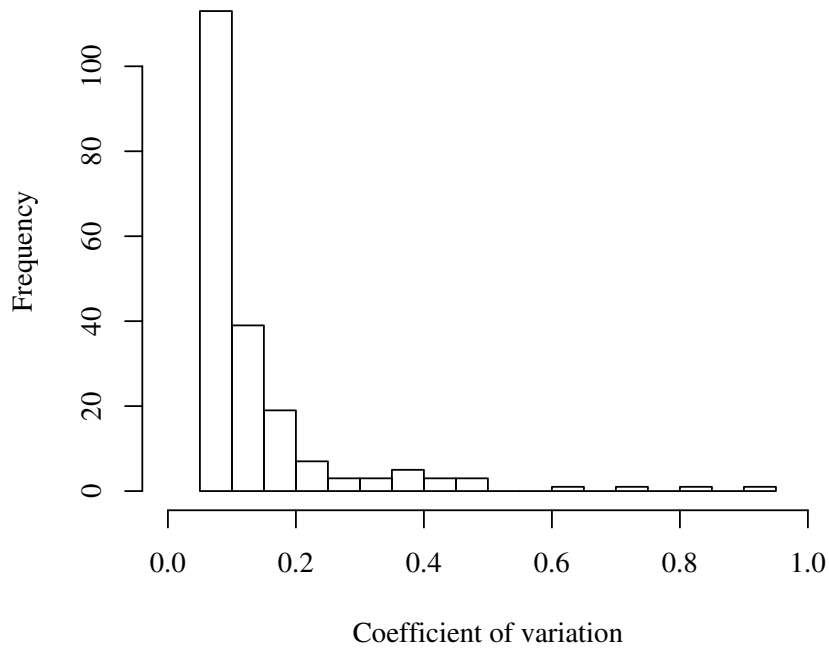


Figure A2. Distribution of the standard errors of the estimated shadow prices of labor



Note: Only coefficients of variation of positive shadow prices are shown.

Figure A3. Coefficients of variation of the estimated shadow prices of labor

Table A6. Estimation Results of the Unrestricted 2nd-Stage Profit Function

Parameter	$i = c$		$i = a$		$i = v$		$i = L$	
	Coef.	(t-val)	Coef.	(t-val)	Coef.	(t-val)	Coef.	(t-val)
α_i	-28774	(-3.22)	32491	(2.05)	-6714	(-0.57)	-62854	(-12.61)
β_{ic}	879	(0.02)	95377	(2.76)	-61671	(-1.14)	-34585	(-4.22)
β_{ia}	95377	(2.76)	76676	(1.19)	-162987	(-2.97)	-9066	(-0.63)
β_{iv}	-61671	(-1.14)	-162987	(-2.97)	221688	(2.95)	2970	(0.24)
β_{iL}	-34585	(-4.22)	-9066	(-0.63)	2970	(0.24)	40681	(7.48)
δ_{ig}	6896	(11.68)	131	(0.12)	-6000	(-7.02)	-3158	(-8.95)
δ_{ik}	0.121	(9.02)	0.292	(12.21)	-0.166	(-9.31)	$7.41 \cdot 10^{-3}$	(0.93)
γ_{gg}	-173	(-3.55)						
γ_{gk}	$9.88 \cdot 10^{-3}$	(9.24)						
γ_{kk}	$-3.55 \cdot 10^{-7}$	(-24.28)						
R^2	0.746		0.494		0.821		0.283	

Note: For definitions of the estimated coefficients see equation (15) of the main article, where the subscripts c , a , v , L , g , and k indicate crop products, animal products, variable inputs, labor, land, and capital, respectively. The standard errors of the coefficients that have not been directly estimated are calculated with the formula of Klein (1953, p. 258). Monotonicity is fulfilled at 98.0% of the observations. The estimation results with convexity imposed are presented in the main article, table 2.

Table A7. Estimation Results of the 2nd-Stage Profit Function with Convexity Imposed

Parameter	$i = c$		$i = a$		$i = v$		$i = L$	
	Coef.	(t-val)	Coef.	(t-val)	Coef.	(t-val)	Coef.	(t-val)
α_i	-31261	(-2.31)	33699	(2.07)	-5480	(-0.37)	-62939	(-6.95)
β_{ic}	53083	(1.86)	64866	(2.75)	-84580	(-2.13)	-33368	(-3.46)
β_{ia}	64866	(2.75)	116773	(2.47)	-168328	(-2.68)	-13311	(-0.63)
β_{iv}	-84580	(-2.13)	-168328	(-2.68)	247344	(2.72)	5564	(0.32)
β_{iL}	-33368	(-3.46)	-13311	(-0.63)	5564	(0.32)	41115	(6.28)
δ_{ig}	6815	(4.59)	303	(0.14)	-6087	(-4.04)	-3181	(-2.81)
δ_{ik}	0.124	(4.40)	0.291	(7.49)	-0.167	(-6.97)	$7.87 \cdot 10^{-3}$	(0.20)
γ_{gg}	-172	(-1.28)						
γ_{gk}	$9.84 \cdot 10^{-3}$	(2.09)						
γ_{kk}	$-3.55 \cdot 10^{-7}$	(-2.26)						
R^2	0.747		0.492		0.821		0.278	

Note: For definitions of the estimated coefficients see equation (15) of the main article, where the subscripts c , a , v , L , g , and k indicate crop products, animal products, variable inputs, labor, land, and capital, respectively. The standard errors of the coefficients are calculated using the bootstrap resampling method (Efron 1979; Efron and Tibshirani 1993). Monotonicity is fulfilled at 97.0% of the observations. The R^2 values are almost identical to the model without convexity imposed, indicating that the data do not unreasonably contradict the convexity constraint (see table A6).

Table A8. Price Elasticities of the Restricted 2nd-Stage Profit Function

	P_c		P_a		P_v		P_L	
	Coef.	(t-val.)	Coef.	(t-val.)	Coef.	(t-val.)	Coef.	(t-val.)
X_c	0.429	(1.99)	0.503	(2.90)	-0.567	(-2.03)	-0.364	(-3.77)
X_a	0.320	(2.90)	0.533	(2.49)	-0.735	(-2.62)	-0.117	(-0.88)
X_v	0.356	(2.03)	0.726	(2.62)	-1.081	(-2.69)	-0.001	(-0.01)
X_L	0.340	(3.77)	0.172	(0.88)	-0.002	(-0.01)	-0.511	(-6.29)

Table A9. Estimation Results of the AIDS

Parameter	$i = m$		$i = a$		$i = L$	
	Coef.	(t-val.)	Coef.	(t-val.)	Coef.	(t-val.)
α_i	0.555	(9.86)	0.185	(14.79)	0.260	(4.18)
β_i	-0.170	(-9.15)	-0.031	(-7.36)	0.201	(9.95)
γ_{im}	0.034	(1.28)	0.021	(0.79)	-0.055	(-5.34)
γ_{ia}	0.021	(0.79)	0.010	(0.35)	-0.031	(-9.36)
γ_{iL}	-0.055	(-5.34)	-0.031	(-9.36)	0.086	(7.97)
R^2	0.409		0.585		0.504	

Note: For definitions of the estimated coefficients see equation (16), where the subscripts m , a , and L indicate purchased market goods, self-produced goods, and leisure, respectively. The standard errors of the coefficients that have not been directly estimated are calculated with the formula of Klein (1953, p. 258). α_0 is set to 10.8, because this value gives the highest likelihood value of the AIDS Model. Monotonicity is fulfilled at 99.5% of the observations and concavity is fulfilled at 88.4% of the observations.

Table A10. Price and Income Elasticities of the AIDS Model

	P_m		P_a		P_L	
	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)
Hicksian Price Elasticities						
C_m	-0.554	(-5.67)	0.144	(1.53)	0.409	(8.59)
C_a	0.648	(1.55)	-0.782	(-1.80)	0.134	(2.55)
C_L	0.176	(8.58)	0.014	(2.77)	-0.190	(-8.53)
Marshallian Price Elasticities						
C_m	-0.667	(-6.80)	0.119	(1.26)	0.149	(2.09)
C_a	0.503	(1.20)	-0.814	(-1.88)	-0.200	(-2.62)
C_L	-0.194	(-9.46)	-0.070	(-13.34)	-1.045	(-31.28)
Income Elasticities						
Y	0.399	(6.08)	0.511	(7.70)	1.308	(42.25)

Labor Market Estimations

The analysis of labor supply and demand of the households is summarized in table 4 of the main article. The bivariate probit estimation shows that labor demand and supply decisions are not significantly correlated in the sample (ρ is not significantly different from zero). The probability that a household supplies off-farm labor increases significantly with the number of household members of working age (N_w) and with the rural nature of the region (W_r).

The probability that a household demands labor significantly depends on the capital endowment (R_k), the endowment of family labor (N_w, N_o), the age of the head of the household (A_h, A_h^2), and the rural nature of the region (W_r). As expected, the probability increases with the capital endowment and decreases with the endowment of family labor. We also observe the expected signs for the age and squared age of the household head, i.e. we observe a u-shaped relation between age and the probability to hire on-farm labor with the lowest probability at the age of 44.4 years. Furthermore, the probability to hire labor decreases with the rural nature of the region.

The effective off-farm wage is significantly influenced by the proportion of supplied labor (X_L^s/T_L), the number of family members of working age (N_w), the age of the head of the household (A_h, A_h^2), and the rural nature of the region (W_r). Larger households and those in more rural areas receive a significantly lower effective off-farm wage. The coefficients of the age and squared age of the household head have the expected signs; i.e. we observe an inverse u-shaped relation between age and the effective off-farm wage with the highest wage at the age of 44.2 years. The estimated parameter of the inverse Mill's ratio is not significantly different from zero, indicating that there is no sample selection bias. If an average household (see table A2 of the main article) increases the amount of supplied labor by 1%, the marginal revenue decreases by 0.075%. If this household doubles the amount of supplied labor from 446 to 892 hours per year, the marginal revenue decreases from 38498 to 35618 PLZ per hour.

The effective on-farm wage is significantly influenced by the amount of hired labor (X_L^h), the capital intensity on the farm (R_k/R_g), the regional unemployment rate (W_u), the regional density of the road and railroad network (W_i), and the rurality of the region (W_r). As expected, farms with a higher degree of mechanization pay higher wages because better skills are required on these farms. The negative impact of the rural nature and the positive impact of the road and railroad network on the effective on-farm wage might reflect heterogeneity of the average regional wages that is not captured in the regional data published by the statistical office (\tilde{P}_L). The positive effect of the regional unemployment rate is counter-intuitive. However, it might be correlated with some other regional variable not included in the analysis. In contrast to the labor supply side, the estimated parameter of the inverse Mill's ratio is significantly different from zero, indicating that an OLS estimation for labor-hiring households would be biased due to non-random sample selection. If an average household (see table A2 of the main article) increases the amount of hired labor by 1%, the marginal cost increases by 0.259%. If this household doubles the amount of hired labor from 211 to 422 hours per year, the marginal cost increases from 38498 to 48467 PLZ per hour.

Estimated Farm-Household Elasticities

Elasticities for Different Labor Regimes

Table A11. Price Elasticities of the Separable FHM (Calculated at Average Values of All Households)

	P_c		P_a		P_v		P_L		P_m	
	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)
X_c	0.43	(1.99)	0.50	(2.90)	-0.57	(-2.03)	-0.36	(-3.77)	0.00	
X_a	0.32	(2.90)	0.53	(2.49)	-0.73	(-2.62)	-0.12	(-0.88)	0.00	
X_v	0.36	(2.03)	0.73	(2.62)	-1.08	(-2.69)	-0.00	(-0.01)	0.00	
X_L	0.34	(3.77)	0.17	(0.88)	-0.00	(-0.01)	-0.51	(-6.29)	0.00	
C_m	0.13	(6.08)	0.33	(3.26)	-0.21	(-6.08)	0.45	(4.20)	-0.67	(-6.80)
C_a	0.17	(7.70)	-0.55	(-1.25)	-0.27	(-7.70)	0.18	(0.41)	0.50	(1.20)
C_L	0.43	(42.25)	0.61	(39.18)	-0.69	(-42.25)	-0.07	(-3.22)	-0.19	(-9.46)
X_L^n	-19.15	(-13.18)	-22.20	(-7.11)	22.00	(9.08)	10.30	(7.07)	6.16	(9.46)
X_L^f	0.34	(3.77)	0.17	(0.88)	-0.00	(-0.01)	-0.51	(-6.29)	0.00	
P_L^*	0.00		0.00		0.00		1.00		0.00	

Table A12. Price Elasticities of the Non-separable FHM for Households that Both Supply and Hire Labor (Calculated at Average Values of All Households)

	P_c		P_a		P_v		P_m	
	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)
X_c	0.28	(1.51)	0.33	(2.06)	-0.39	(-1.53)	0.05	(2.67)
X_a	0.27	(2.40)	0.48	(2.30)	-0.68	(-2.26)	0.02	(0.87)
X_v	0.36	(2.10)	0.73	(2.57)	-1.08	(-2.61)	0.00	(0.01)
X_L	0.13	(1.43)	-0.08	(-0.50)	0.24	(1.98)	0.07	(3.32)
C_m	0.30	(6.21)	0.53	(4.76)	-0.41	(-6.74)	-0.72	(-7.32)
C_a	0.23	(7.53)	-0.48	(-1.13)	-0.33	(-8.25)	0.48	(1.16)
C_L	0.35	(15.54)	0.52	(18.27)	-0.60	(-21.22)	-0.17	(-7.98)
X_L^h	1.52	(1.46)	1.76	(1.26)	-1.75	(-1.37)	-0.49	(-1.26)
X_L^s	-6.26	(-3.79)	-7.25	(-3.56)	7.19	(3.69)	2.01	(3.55)
X_L^n	-13.25	(-3.46)	-15.37	(-3.42)	15.23	(3.45)	4.26	(3.42)
X_L^f	0.04	(0.16)	-0.19	(-0.60)	0.37	(1.22)	0.10	(1.28)
P_L^*	0.42	(3.94)	0.49	(3.68)	-0.48	(-3.82)	-0.13	(-3.67)

Table A13. Price Elasticities of the Non-separable FHM for Households that Both Supply and Hire Labor (Calculated at Average Values of Households in this Labor Regime)

	P_c		P_a		P_v		P_m	
	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)
X_c	0.28	(1.53)	0.33	(2.09)	-0.40	(-1.55)	0.05	(2.56)
X_a	0.27	(2.44)	0.48	(2.31)	-0.68	(-2.27)	0.02	(0.86)
X_v	0.36	(2.10)	0.73	(2.57)	-1.08	(-2.62)	0.00	(0.01)
X_L	0.14	(1.51)	-0.06	(-0.41)	0.23	(1.84)	0.07	(3.11)
C_m	0.30	(5.87)	0.52	(4.64)	-0.40	(-6.42)	-0.72	(-7.28)
C_a	0.22	(7.50)	-0.49	(-1.13)	-0.33	(-8.20)	0.49	(1.17)
C_L	0.36	(15.23)	0.52	(17.96)	-0.60	(-20.81)	-0.17	(-7.99)
X_L^h	1.30	(1.35)	1.51	(1.18)	-1.50	(-1.27)	-0.42	(-1.18)
X_L^s	-5.47	(-3.46)	-6.34	(-3.28)	6.29	(3.38)	1.76	(3.28)
X_L^n	-13.41	(-3.12)	-15.55	(-3.10)	15.41	(3.11)	4.31	(3.10)
X_L^f	0.04	(0.14)	-0.19	(-0.55)	0.37	(1.12)	0.11	(1.17)
P_L^*	0.40	(3.60)	0.46	(3.39)	-0.46	(-3.51)	-0.13	(-3.40)

Note: To focus on the effect of the labor market regime, only X_L^s , X_L^h , z^s and z^h are the average values of households in this labor regime, while X_c , X_a , X_v , X_L , C_m , X_a and C_L are taken from the whole sample. $X_L^f = X_L - X_L^H$ and $T_L = X_L^S + X_L^F + C_L$ are calculated residually.

Table A14. Price Elasticities of the Non-separable FHM for Households that only Supply Labor (Calculated at Average Values of Households in this Labor Regime)

	P_c		P_a		P_v		P_m	
	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)
X_c	0.24	(1.05)	0.29	(1.23)	-0.35	(-1.16)	0.06	(1.13)
X_a	0.26	(2.06)	0.46	(2.14)	-0.67	(-2.14)	0.02	(0.71)
X_v	0.36	(2.10)	0.73	(2.55)	-1.08	(-2.60)	0.00	(0.01)
X_L	0.08	(0.35)	-0.13	(-0.46)	0.30	(1.10)	0.08	(1.17)
C_m	0.34	(1.93)	0.57	(2.52)	-0.45	(-2.20)	-0.73	(-6.51)
C_a	0.24	(3.69)	-0.47	(-1.09)	-0.35	(-4.52)	0.48	(1.16)
C_L	0.34	(4.09)	0.50	(5.20)	-0.58	(-6.08)	-0.16	(-4.94)
X_L^s	-2.22	(-1.16)	-2.58	(-1.15)	2.55	(1.16)	0.72	(1.15)
X_L^f	0.08	(0.10)	-0.13	(-0.14)	0.30	(0.32)	0.08	(0.32)
P_L^*	0.51	(1.19)	0.59	(1.18)	-0.59	(-1.19)	-0.16	(-1.18)

Note: see note below table [A13](#).

Table A15. Price Elasticities of the Non-separable FHM for Households that only Hire Labor (Calculated at Average Values of Households in this Labor Regime)

	P_c		P_a		P_v		P_m	
	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)
X_c	0.05	(0.33)	0.06	(0.41)	-0.13	(-0.56)	0.12	(3.32)
X_a	0.20	(1.28)	0.39	(1.65)	-0.59	(-1.69)	0.04	(0.91)
X_v	0.36	(1.97)	0.72	(2.37)	-1.08	(-2.46)	0.00	(0.01)
X_L	-0.19	(-2.56)	-0.45	(-4.59)	0.61	(6.94)	0.17	(5.35)
C_m	0.56	(9.20)	0.82	(7.10)	-0.70	(-9.84)	-0.80	(-8.25)
C_a	0.31	(5.71)	-0.38	(-0.92)	-0.43	(-6.70)	0.46	(1.11)
C_L	0.23	(8.30)	0.38	(10.99)	-0.46	(-13.92)	-0.13	(-6.62)
X_L^h	2.42	(0.40)	2.80	(0.40)	-2.78	(-0.40)	-0.78	(-0.40)
X_L^f	-0.54	(-8.30)	-0.88	(-10.99)	1.06	(13.92)	0.30	(6.62)
P_L^*	1.05	(8.01)	1.21	(7.39)	-1.20	(-7.92)	-0.34	(-5.61)

Note: see note below table [A13](#).

Table A16. Price Elasticities of the Non-separable FHM for Autarkic Households (Calculated at Average Values of Households in this Labor Regime)

	P_c		P_a		P_v		P_m	
	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)	Elast.	(t-val.)
X_c	-0.07	(-0.51)	-0.07	(-0.49)	0.00	(0.01)	0.16	(3.40)
X_a	0.16	(0.88)	0.35	(1.32)	-0.55	(-1.44)	0.05	(0.92)
X_v	0.35	(1.84)	0.72	(2.24)	-1.08	(-2.37)	0.00	(0.01)
X_L	-0.35	(-5.77)	-0.63	(-9.15)	0.80	(11.88)	0.22	(6.00)
C_m	0.69	(10.50)	0.97	(8.51)	-0.85	(-11.88)	-0.85	(-8.75)
C_a	0.35	(5.16)	-0.34	(-0.82)	-0.48	(-6.07)	0.44	(1.08)
C_L	0.17	(5.77)	0.31	(9.15)	-0.39	(-11.88)	-0.11	(-6.00)
X_L^f	-0.35	(-5.77)	-0.63	(-9.15)	0.80	(11.88)	0.22	(6.00)
P_L^*	1.36	(9.17)	1.58	(9.44)	-1.56	(-9.79)	-0.44	(-5.65)

Note: see note below table [A13](#).

Table A17. Differences between Price Elasticities of the Separable FHM and the Non-separable FHM for Households that Supply as well as Demand Labor (Calculated at Average Values of All Households)

	P_c		P_a		P_v		P_m	
	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)
X_c	0.15	(2.43)	0.18	(2.73)	-0.18	(-2.55)	-0.05	(-2.67)
X_a	0.05	(0.87)	0.06	(0.78)	-0.06	(-0.81)	-0.02	(-0.87)
X_v	0.00	(0.01)	0.00	(0.01)	-0.00	(-0.01)	-0.00	(-0.01)
X_L	0.21	(3.37)	0.25	(2.85)	-0.25	(-3.08)	-0.07	(-3.32)
C_m	-0.17	(-3.64)	-0.20	(-3.42)	0.20	(3.54)	0.06	(4.02)
C_a	-0.06	(-2.16)	-0.07	(-2.09)	0.06	(2.14)	0.02	(2.19)
C_L	0.08	(3.64)	0.09	(3.41)	-0.09	(-3.54)	-0.03	(-4.00)
X_L^n	-5.90	(-1.50)	-6.84	(-1.38)	6.77	(1.44)	1.90	(1.47)
X_L^f	0.30	(1.21)	0.36	(1.24)	-0.37	(-1.28)	-0.10	(-1.28)
P_L^*	-0.42	(-3.94)	-0.49	(-3.68)	0.48	(3.82)	0.13	(3.67)

Table A18. Differences between Price Elasticities of the Separable FHM and the Non-separable FHM for Households that Supply as well as Demand Labor

	P_c		P_a		P_v		P_m	
	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)
X_c	0.15	(2.34)	0.17	(2.60)	-0.17	(-2.45)	-0.05	(-2.56)
X_a	0.05	(0.87)	0.05	(0.78)	-0.05	(-0.80)	-0.02	(-0.86)
X_v	0.00	(0.01)	0.00	(0.01)	-0.00	(-0.01)	-0.00	(-0.01)
X_L	0.20	(3.14)	0.24	(2.71)	-0.23	(-2.90)	-0.07	(-3.11)
C_m	-0.16	(-3.37)	-0.19	(-3.19)	0.19	(3.29)	0.05	(3.66)
C_a	-0.05	(-2.10)	-0.06	(-2.03)	0.06	(2.08)	0.02	(2.13)
C_L	0.08	(3.37)	0.09	(3.18)	-0.09	(-3.29)	-0.02	(-3.65)
X_L^n	-5.74	(-1.30)	-6.66	(-1.21)	6.60	(1.26)	1.85	(1.28)
X_L^f	0.30	(1.08)	0.37	(1.13)	-0.38	(-1.17)	-0.11	(-1.17)
P_L^*	-0.40	(-3.60)	-0.46	(-3.39)	0.46	(3.51)	0.13	(3.40)

Table A19. Differences between Price Elasticities of the Separable FHM and the Non-separable FHM for Households that only Supply Labor

	P_c		P_a		P_v		P_m	
	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)
X_c	0.19	(1.11)	0.22	(1.14)	-0.21	(-1.12)	-0.06	(-1.13)
X_a	0.06	(0.72)	0.07	(0.66)	-0.07	(-0.68)	-0.02	(-0.71)
X_v	0.00	(0.01)	0.00	(0.01)	-0.00	(-0.01)	-0.00	(-0.01)
X_L	0.26	(1.17)	0.30	(1.15)	-0.30	(-1.16)	-0.08	(-1.17)
C_m	-0.21	(-1.18)	-0.24	(-1.17)	0.24	(1.18)	0.07	(1.19)
C_a	-0.07	(-1.08)	-0.08	(-1.07)	0.08	(1.08)	0.02	(1.08)
C_L	0.10	(1.18)	0.11	(1.17)	-0.11	(-1.18)	-0.03	(-1.19)
X_L^n	-16.93	(-7.30)	-19.62	(-5.46)	19.45	(6.26)	5.45	(6.53)
X_L^f	0.26	(0.32)	0.30	(0.32)	-0.30	(-0.32)	-0.08	(-0.32)
P_L^*	-0.51	(-1.19)	-0.59	(-1.18)	0.59	(1.19)	0.16	(1.18)

Table A20. Differences between Price Elasticities of the Separable FHM and the Non-separable FHM for Households that only Demand Labor

	P_c		P_a		P_v		P_m	
	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)
X_c	0.38	(2.97)	0.44	(3.73)	-0.44	(-3.26)	-0.12	(-3.32)
X_a	0.12	(0.92)	0.14	(0.81)	-0.14	(-0.84)	-0.04	(-0.91)
X_v	0.00	(0.01)	0.00	(0.01)	-0.00	(-0.01)	-0.00	(-0.01)
X_L	0.53	(6.19)	0.62	(4.30)	-0.61	(-5.01)	-0.17	(-5.35)
C_m	-0.43	(-6.86)	-0.50	(-6.38)	0.49	(6.75)	0.14	(8.39)
C_a	-0.14	(-2.52)	-0.16	(-2.45)	0.16	(2.51)	0.05	(2.52)
C_L	0.20	(6.88)	0.23	(6.35)	-0.23	(-6.76)	-0.06	(-8.27)
X_L^n	-21.57	(-3.43)	-25.00	(-3.23)	24.78	(3.34)	6.94	(3.35)
X_L^f	0.88	(9.79)	1.05	(6.42)	-1.06	(-7.87)	-0.30	(-6.62)
P_L^*	-1.05	(-8.01)	-1.21	(-7.39)	1.20	(7.92)	0.34	(5.61)

Table A21. Differences between Price Elasticities of the Separable FHM and the Non-separable FHM for Autarkic Households

	P_c		P_a		P_v		P_m	
	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)
X_c	0.50	(3.06)	0.57	(4.04)	-0.57	(-3.43)	-0.16	(-3.40)
X_a	0.16	(0.93)	0.19	(0.82)	-0.18	(-0.85)	-0.05	(-0.92)
X_v	0.00	(0.01)	0.00	(0.01)	-0.00	(-0.01)	-0.00	(-0.01)
X_L	0.69	(7.97)	0.81	(4.99)	-0.80	(-6.00)	-0.22	(-6.00)
C_m	-0.56	(-8.19)	-0.65	(-8.22)	0.64	(8.52)	0.18	(9.49)
C_a	-0.18	(-2.58)	-0.21	(-2.54)	0.21	(2.59)	0.06	(2.55)
C_L	0.26	(8.24)	0.30	(8.17)	-0.30	(-8.57)	-0.08	(-9.36)
X_L^n	-19.15	(-13.18)	-22.20	(-7.11)	22.00	(9.08)	6.16	(9.46)
X_L^f	0.69	(7.97)	0.81	(4.99)	-0.80	(-6.00)	-0.22	(-6.00)
P_L^*	-1.36	(-9.17)	-1.58	(-9.44)	1.56	(9.79)	0.44	(5.65)

Table A22. Differences between Price Elasticities of the Households that Supply as well as Demand Labor and the Households that only Supply Labor

	P_c		P_a		P_v		P_m	
	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)
X_c	0.04	(0.34)	0.05	(0.34)	-0.05	(-0.34)	-0.01	(-0.34)
X_a	0.01	(0.32)	0.01	(0.32)	-0.01	(-0.32)	-0.00	(-0.32)
X_v	0.00	(0.01)	0.00	(0.01)	-0.00	(-0.01)	-0.00	(-0.01)
X_L	0.06	(0.34)	0.07	(0.34)	-0.06	(-0.34)	-0.02	(-0.34)
C_m	-0.05	(-0.34)	-0.05	(-0.34)	0.05	(0.34)	0.01	(0.34)
C_a	-0.01	(-0.34)	-0.02	(-0.34)	0.02	(0.34)	0.00	(0.34)
C_L	0.02	(0.34)	0.02	(0.34)	-0.02	(-0.34)	-0.01	(-0.34)
X_L^h	1.30	(0.47)	1.51	(0.47)	-1.50	(-0.47)	-0.42	(-0.47)
X_L^s	-3.25	(-6.98)	-3.77	(-5.67)	3.73	(6.32)	1.05	(5.84)
X_L^n	-11.19	(-4.70)	-12.97	(-4.65)	12.85	(4.68)	3.60	(4.66)
X_L^f	-0.04	(-0.07)	-0.06	(-0.10)	0.08	(0.12)	0.02	(0.12)
P_L^*	-0.11	(-0.34)	-0.13	(-0.34)	0.13	(0.34)	0.04	(0.34)

Table A23. Differences between Price Elasticities of the Households that Supply as well as Demand Labor and the Households that only Demand Labor

	P_c		P_a		P_v		P_m	
	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)
X_c	0.24	(2.59)	0.27	(3.12)	-0.27	(-2.81)	-0.08	(-2.77)
X_a	0.08	(0.92)	0.09	(0.81)	-0.09	(-0.84)	-0.02	(-0.91)
X_v	0.00	(0.01)	0.00	(0.01)	-0.00	(-0.01)	-0.00	(-0.01)
X_L	0.33	(4.17)	0.38	(3.52)	-0.38	(-3.81)	-0.11	(-3.77)
C_m	-0.26	(-4.17)	-0.31	(-4.20)	0.30	(4.23)	0.09	(4.28)
C_a	-0.09	(-2.28)	-0.10	(-2.25)	0.10	(2.29)	0.03	(2.25)
C_L	0.12	(4.17)	0.14	(4.20)	-0.14	(-4.23)	-0.04	(-4.27)
X_L^h	-1.11	(-0.21)	-1.29	(-0.21)	1.28	(0.21)	0.36	(0.21)
X_L^s	-5.47	(-3.46)	-6.34	(-3.28)	6.29	(3.38)	1.76	(3.28)
X_L^n	-15.83	(-1.53)	-18.35	(-1.53)	18.18	(1.53)	5.09	(1.53)
X_L^f	0.58	(2.13)	0.68	(2.11)	-0.69	(-2.17)	-0.19	(-2.11)
P_L^*	-0.65	(-4.25)	-0.75	(-4.31)	0.74	(4.33)	0.21	(3.64)

Table A24. Differences between Price Elasticities of the Households that Supply as well as Demand Labor and the Autarkic Households

	P_c		P_a		P_v		P_m	
	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)
X_c	0.35	(2.88)	0.41	(3.76)	-0.40	(-3.22)	-0.11	(-3.09)
X_a	0.11	(0.94)	0.13	(0.83)	-0.13	(-0.86)	-0.04	(-0.93)
X_v	0.00	(0.01)	0.00	(0.01)	-0.00	(-0.01)	-0.00	(-0.01)
X_L	0.49	(6.25)	0.57	(4.68)	-0.56	(-5.32)	-0.16	(-4.89)
C_m	-0.39	(-5.79)	-0.46	(-6.27)	0.45	(6.15)	0.13	(5.76)
C_a	-0.13	(-2.47)	-0.15	(-2.47)	0.15	(2.50)	0.04	(2.42)
C_L	0.18	(5.82)	0.21	(6.26)	-0.21	(-6.19)	-0.06	(-5.74)
X_L^h	1.30	(1.35)	1.51	(1.18)	-1.50	(-1.27)	-0.42	(-1.18)
X_L^s	-5.47	(-3.46)	-6.34	(-3.28)	6.29	(3.38)	1.76	(3.28)
X_L^n	-13.41	(-3.12)	-15.55	(-3.10)	15.41	(3.11)	4.31	(3.10)
X_L^f	0.40	(1.41)	0.44	(1.32)	-0.42	(-1.30)	-0.12	(-1.30)
P_L^*	-0.96	(-5.76)	-1.11	(-6.30)	1.10	(6.15)	0.31	(4.32)

Table A25. Differences between Price Elasticities of the Households that only Supply Labor and the Households that only Demand Labor

	P_c		P_a		P_v		P_m	
	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)
X_c	0.20	(1.14)	0.23	(1.18)	-0.22	(-1.16)	-0.06	(-1.16)
X_a	0.06	(0.75)	0.07	(0.69)	-0.07	(-0.71)	-0.02	(-0.74)
X_v	0.00	(0.01)	0.00	(0.01)	-0.00	(-0.01)	-0.00	(-0.01)
X_L	0.27	(1.22)	0.32	(1.20)	-0.31	(-1.21)	-0.09	(-1.21)
C_m	-0.22	(-1.22)	-0.25	(-1.22)	0.25	(1.22)	0.07	(1.22)
C_a	-0.07	(-1.11)	-0.08	(-1.11)	0.08	(1.11)	0.02	(1.11)
C_L	0.10	(1.22)	0.12	(1.22)	-0.12	(-1.22)	-0.03	(-1.22)
X_L^h	-2.42	(-0.30)	-2.80	(-0.30)	2.78	(0.30)	0.78	(0.30)
X_L^s	-2.22	(-1.16)	-2.58	(-1.15)	2.55	(1.16)	0.72	(1.15)
X_L^n	-4.64	(-0.58)	-5.38	(-0.58)	5.33	(0.58)	1.49	(0.58)
X_L^f	0.62	(0.76)	0.75	(0.79)	-0.76	(-0.82)	-0.21	(-0.81)
P_L^*	-0.54	(-1.22)	-0.62	(-1.22)	0.62	(1.22)	0.17	(1.20)

Table A26. Differences between Price Elasticities of the Households that only Supply Labor and the Autarkic Households

	P_c		P_a		P_v		P_m	
	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)
X_c	0.31	(1.67)	0.36	(1.80)	-0.36	(-1.73)	-0.10	(-1.70)
X_a	0.10	(0.86)	0.12	(0.77)	-0.11	(-0.80)	-0.03	(-0.85)
X_v	0.00	(0.01)	0.00	(0.01)	-0.00	(-0.01)	-0.00	(-0.01)
X_L	0.43	(1.94)	0.50	(1.88)	-0.50	(-1.91)	-0.14	(-1.88)
C_m	-0.35	(-1.92)	-0.40	(-1.94)	0.40	(1.94)	0.11	(1.92)
C_a	-0.11	(-1.57)	-0.13	(-1.58)	0.13	(1.58)	0.04	(1.56)
C_L	0.16	(1.93)	0.19	(1.94)	-0.19	(-1.94)	-0.05	(-1.92)
X_L^s	-2.22	(-1.16)	-2.58	(-1.15)	2.55	(1.16)	0.72	(1.15)
X_L^n	-2.22	(-1.16)	-2.58	(-1.15)	2.55	(1.16)	0.72	(1.15)
X_L^f	0.43	(0.53)	0.50	(0.53)	-0.50	(-0.53)	-0.14	(-0.53)
P_L^*	-0.85	(-1.92)	-0.99	(-1.94)	0.98	(1.94)	0.27	(1.84)

Table A27. Differences between Price Elasticities of the Households that only Demand Labor and the Autarkic Households

	P_c		P_a		P_v		P_m	
	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)	Diff.	(t-val.)
X_c	0.11	(2.28)	0.13	(2.72)	-0.13	(-2.47)	-0.04	(-2.34)
X_a	0.04	(0.94)	0.04	(0.83)	-0.04	(-0.86)	-0.01	(-0.92)
X_v	0.00	(0.01)	0.00	(0.01)	-0.00	(-0.01)	-0.00	(-0.01)
X_L	0.16	(3.32)	0.19	(3.10)	-0.18	(-3.22)	-0.05	(-2.97)
C_m	-0.13	(-3.11)	-0.15	(-3.30)	0.15	(3.23)	0.04	(3.01)
C_a	-0.04	(-2.07)	-0.05	(-2.09)	0.05	(2.10)	0.01	(2.01)
C_L	0.06	(3.12)	0.07	(3.30)	-0.07	(-3.23)	-0.02	(-3.01)
X_L^h	2.42	(0.40)	2.80	(0.40)	-2.78	(-0.40)	-0.78	(-0.40)
X_L^n	2.42	(0.40)	2.80	(0.40)	-2.78	(-0.40)	-0.78	(-0.40)
X_L^f	-0.18	(-4.46)	-0.24	(-5.22)	0.26	(5.63)	0.07	(4.74)
P_L^*	-0.31	(-3.03)	-0.36	(-3.21)	0.36	(3.14)	0.10	(2.68)

References

- Abdulai, A., and C.L. Delgado. 1999. "Determinants of Nonfarm Earnings of Farm-Based Husbands and Wives in Northern Ghana." *American Journal of Agricultural Economics* 81:117–130.
- Abdulai, A., P. Monnin, and J. Gerber. 2005. "Output Uncertainty, Information Acquisition and the Adoption of New Technologies." Unpublished Working Paper, University of Kiel, Germany.
- Benjamin, D. 1992. "Household Composition, Labor Market, and Labor Demand: Testing for Separation in Agricultural Household Models." *Econometrica* 60:287–322.
- Blundell, R.W., and J.M. Robin. 1999. "Estimation in Large and Disaggregated Demand Systems: An Estimator for Conditionally Linear Systems." *Journal of Applied Econometrics* 14:209–232.
- Carter, M.R., and P. Olinto. 2003. "Getting Institutions "Right" for Whom? Credit Constraints and the Impact of Property Rights on the Quantity and Composition of Investment." *American Journal of Agricultural Economics* 85:173–186.
- Carter, M.R., and Y. Yao. 2002. "Local versus Global Separability in Agricultural Household Models: The Factor Price Equalization Effect of Land Transfer Rights." *American Journal of Agricultural Economics* 84:702–715.
- Chambers, R.G., and R.E. Lopez. 1987. "Tax Policies and the Financially Constrained Farm Household." *American Journal of Agricultural Economics* 69:369–377.
- de Janvry, A., M. Fafchamps, and E. Sadoulet. 1991. "Peasant Household Behaviour with Missing Markets: Some Paradoxes Explained." *The Economic Journal* 101:1400–1417.
- de Janvry, A., E. Sadoulet, M. Fafchamps, and M. Raki. 1992. "Structural Adjustment and the Peasantry in Morocco: A Computable Household Model." *European Review of Agricultural Economics* 19:427–453.
- Deaton, A., and J. Muellbauer. 1980. "An Almost Ideal Demand System." *The American Economic Review* 70:312–326.
- Deolalikar, A.B., and W.P.M. Vijverberg. 1983. "The Heterogeneity of Family and Hired Labour in Agricultural Production: A Test Using District-level Data from India." *Journal of Economic Development* 8:45–69.
- . 1987. "A Test of Heterogeneity of Family and Hired Labour in Asian Agriculture." *Oxford Bulletin of Economics and Statistics* 49:291–305.
- Diewert, W.E., and T.J. Wales. 1987. "Flexible Functional Forms and Global Curvature Conditions." *Econometrica* 55:43–68.
- . 1992. "Quadratic Spline Models for Producer's Supply and Demand Functions." *International Economic Review* 33:705–722.

- Efron, B. 1979. "Bootstrap Methods: Another Look at the Jackknife." *The Annals of Statistics* 7:1–26.
- Efron, B., and R.J. Tibshirani. 1993. *An Introduction to the Bootstrap*, vol. 57 of *Monographs on Statistics and Applied Probability*. New York, London: Chapman & Hall.
- Eswaran, M., and A. Kotwal. 1986. "Access to Capital and Agrarian Production Organization." *The Economic Journal* 96:482–498.
- Fafchamps, M. 1992. "Cash Crop Production, Food Price Volatility, and Rural Market Integration in the Third World." *American Journal of Agricultural Economics* 74:90–99.
- Feder, G. 1985. "The Relation between Farm Size and Farm Productivity: The Role of Family Labor, Supervision and Credit Constraints." *Journal of Development Economics* 18:297–313.
- Finkelshtain, I., and J.A. Chalfant. 1991. "Marketed Surplus under Risk: Do Peasants Agree with Sandmo?" *American Journal of Agricultural Economics* 73:557–567.
- Frisvold, G.B. 1994. "Does Supervision Matter? Some Hypothesis Tests Using Indian Farm-Level Data." *Journal of Development Economics* 43:217–238.
- Genz, A., F. Bretz, and T. Hothorn. 2005. *mvtnorm: Multivariate Normal and t Distribution*. R package version 0.7-2, <http://cran.r-project.org>.
- Genz, A., M. Meyer, T. Lumley, and M. Maechler. 2005. *adapt: Multidimensional Numerical Integration*. R package version 1.0-3, <http://cran.r-project.org>.
- Glauben, T., C.H.C.A. Henning, and A. Henningsen. 2003. "Farm household decisions under various tax policies: Comparative static results and evidence from household data." Paper presented at the 25th International Conference of the IAAE, Durban, South Africa.
- Główny Urząd Statystyczny. 1996. "Informacja o Sytuacji Społeczno - Gospodarczej Województw." *Kwartalnik* Nr 1-2, Warsaw.
- Goetz, S.J. 1992. "A Selectivity Model of Household Food Marketing Behavior in Sub-Saharan Africa." *American Journal of Agricultural Economics* 74:444–452.
- Gourieroux, C., A. Monfort, and A. Trognon. 1985. "Moindres Carres Asymptotiques." *Annales de l'INSEE* 58:91–122.
- Granovetter, M.S. 1973. "The strength of weak ties." *The American Journal of Sociology* 78:1360–1380.
- Greene, W.H. 2003. *Econometric Analysis*, 5th ed. Prentice Hall.
- Hamann, J.D., and A. Henningsen. 2006. *systemfit: Simultaneous Equation Estimation*. R package version 0.8-0, <http://cran.r-project.org>.
- Henning, C.H.C.A. 1994. *Unternehmens-Haushalts-Modelle. Eine theoretische und empirische Analyse*. Berlin: Duncker & Humblot.
- Henning, C.H.C.A., and A. Henningsen. 2007. "AJAE Appendix: Modeling Farm Households' Price Responses in the Presence of Transaction Costs and Heterogeneity in Labor Markets." Unpublished manuscript. Available at: <http://agecon.lib.umn.edu/>.

- Henningsen, A., and O. Toomet. 2005. *micEcon: Tools for Microeconomic Analysis and Microeconomic Modeling*. R package version 0.2-2, <http://cran.r-project.org>.
- IERiGZ. 1995. "Indywidualne wyniki rachunkowosci rolnej gospodarstw chlopskich - Rok 1991-1994 - Makroregion VIII - Srodkowo Zachodni." Unpublished data, personally requested from IERiGZ, Warsaw, Poland.
- Jacoby, H. 1993. "Shadow Wages and Peasant Family Labour Supply: An Econometric Application to the Peruvian Sierra." *Review of Economic Studies* 60:903–921.
- Key, N., E. Sadoulet, and A. de Janvry. 2000. "Transaction Costs and Agricultural Household Supply Response." *American Journal of Agricultural Economics* 82:245–259.
- Klein, L.R. 1953. *A Textbook of Econometrics*. New York: Row, Peterson and Co.
- Kodde, D.A., F.C. Palm, and G.A. Pfann. 1990. "Asymptotic Least-Squares Estimation Efficiency Considerations and Applications." *Journal of Applied Econometrics* 5:229–243.
- Koebel, B. 1998. "Tests of Representative Firm Models: Results for German Manufacturing Industries." *Journal of Productivity Analysis* 10:251–270.
- Koebel, B., M. Falk, and F. Laisney. 2003. "Imposing and Testing Curvature Conditions on a Box-Cox Cost Function." *Journal of Business and Economic Statistics* 21:319–335.
- Kohli, U.R. 1993. "A Symmetric Normalized Quadratic GNP Function and the US Demand for Imports and Supply of Exports." *International Economic Review* 34:243–255.
- Lau, L.J. 1978a. "Applications of Profit Functions." In M. Fuss and D. McFadden, eds. *Production Economics: A Dual Approach to Theory and Applications, Vol 1*. Amsterdam: North-Holland, pp. 133–216.
- . 1978b. "Testing and Imposing Monotonicity, Convexity and Quasi-Convexity Constraints." In M. Fuss and D. McFadden, eds. *Production Economics: A Dual Approach to Theory and Applications, Vol. 1*. Amsterdam: North-Holland, pp. 409–453.
- Lee, L., G.S. Maddala, and R.P. Trost. 1980. "Asymptotic Covariance Matrices of Two-stage Probit and Two-stage Tobit Methods for Simultaneous Equations Models with Selectivity." *Econometrica* 48:491–503.
- Lopez, R. 1984. "Estimating Labor Supply and Production Decisions of Self-Employed Farm Producers." *European Economic Review* 24:61–82.
- . 1986. "Structural Models of the Farm Household that Allow for Interdependent Utility and Profit Maximization Decisions." In I. Singh, L. Squire, and J. Strauss, eds. *Agricultural Household Models*. Baltimore: The John Hopkins University Press, pp. 306–325.
- Omamo, S.W. 1998. "Transportation Costs and Smallholder Cropping Choices: An Application to Siaya District, Kenya." *American Journal of Agricultural Economics* 80:1083–1095.
- Pagan, A. 1984. "Econometric Issues in the Analysis of Regressions with Generated Regressors." *International Economic Review* 25:221–247.

- R Development Core Team. 2005. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria, ISBN 3-900051-07-0.
- Sadoulet, E., A. de Janvry, and C. Benjamin. 1998. "Household Behavior with Imperfect Labor Markets." *Industrial Relations* 37:85–108.
- . 1996. "Labor Market Imperfections and Selective Separability in Household Models: A Predictive Typology of Mexican Ejidatarios." Working Paper No. 786, Department of Agricultural and Resource Economics, University of California at Berkeley.
- Saha, A., H.A. Love, and R.B. Schwart. 1994. "Adoption of Emerging Technologies under Output Uncertainty." *American Journal of Agricultural Economics* 76:836–846.
- Skoufias, E. 1994. "Using Shadow Wages to Estimate Labor Supply of Agricultural Households." *American Journal of Agricultural Economics* 76:215–227.
- Spence, A.M. 1976. "Consumer Misperceptions, Product Failure and Producer Liability." *Quarterly Journal of Economics* 90:591–597.
- Strauss, J. 1986. "The Theory and Comparative Statics of Agricultural Household Models: A General Approach." In I. Singh, L. Squire, and J. Strauss, eds. *Agricultural Household Models: Extensions, Applications, and Policy*. Baltimore: Johns Hopkins University Press, pp. 71–91.
- Thijssen, G.J. 1988. "Estimating a Labour Supply Function of Farm Households." *European Review of Agricultural Economics* 15:67–78.
- Vakis, R., E. Sadoulet, and A. de Janvry. 2003. "Measuring Transactions Costs from Observed Behavior: Market Choices in Peru." <http://are.berkeley.edu/~sadoulet/papers/TC-wp.pdf>.
- Vance, C., and J. Geoghegan. 2004. "Modeling the Determinants of Semi-Subsistent and Commercial Land Uses in an Agricultural Frontier of Southern Mexico: A Switching Regression Approach." *International Regional Science Review* 27:326–347.
- Venables, W.N., and B.D. Ripley. 2002. *Modern Applied Statistics with S*, 4th ed. New York: Springer.
- Yee, T.W., and C.J. Wild. 1996. "Vector Generalized Additive Models." *Journal of the Royal Statistical Society, Series B, Methodological* 58:481–493.
- Zawadzki, W. 1994. "Infrastruktura Techniczna - Glowny Warunek Wielofunkcyjnego Rozwoju Obszarow Wiejskich." Tech. Rep. 365, IERiGZ, Warsaw.